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SIR DAVID BREWSTER, President, in the Chair.

At the request of the Council, Professor William Thomson of Glasgow delivered the following Address on the Forces concerned in the Laying and Lifting of Deep-Sea Cables.

THE forces concerned in the laying and lifting of deep submarine cables attracted much public attention in the years 1857–58.

An experimental trip to the Bay of Biscay in May 1858, proved the possibility, not only of safely laying such a rope as the old Atlantic cable in very deep water, but of lifting it from the bottom without fracture. The speaker had witnessed the almost incredible feat of lifting up a considerable length of that slight and seemingly fragile thread from a depth of nearly 2½ nautical miles.* The cable had actually brought with it safely to the surface, from the bottom, a splice with a large weighted frame attached to it, to prevent untwisting between the two ships, from which two portions of cable with opposite twists had been laid. The actual laying of the cable a few months later, from mid ocean to Valencia on one side, and Trinity Bay, Newfoundland, on the other, regarded merely as a mechanical achievement, took by surprise some of the most celebrated engineers of the day, who had not concealed their opinion, that the Atlantic Telegraph Cumpany had undertaken an impossible problem. As a mechanical achievement it was completely successful; and the electric failure, after several hundred messages (comprising upwards of 4359 words) had been transmitted between Valencia and Newfoundland, was owing to electric faults existing in the cable before it went to sea. Such faults cannot escape detection, in the course of the manufacture, under the improved electric testing since brought into practice, and the causes which led to the failure of the first Atlantic cable no longer exist as dangers in submarine telegraphic enterprise. But the possibility of damage being done to the insulation of the electric conductor before it leaves the ship (illustrated by the occurrences which led to the temporary loss of the 1865 cable), implies a danger which can only be thoroughly guarded against by being ready at any moment to back the ship and check the egress of the cable, and to hold on for some time, or to haul back some length according to the results of electric testing.

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* Throughout the following statements, the word mile will be used to denote (not that most meaningless of modern measures, the British statute mile) but the nautical mile, or the length of a minute of latitude, in mean latitudes, which is 6073 feet. For approximate statements, rough estimates, &c., it may be taken as 6000 feet, or 1000 fathoms.

The forces concerned in these operations, and the mechanical arrangements by which they are applied and directed, constitute one chief part of the present address; the remainder is devoted to explanations as to the problem of lifting the west end of the 1200 miles of cable laid last summer, from Valencia westwards, and now lying in perfect electric condition (in the very safest place in which a submarine cable can be kept), and ready to do its work, as soon as it is connected with Newfoundland, by the 600 miles required to complete the line.

Forces concerned in the Submergence of a Cable.

In a paper published in the "Engineer" Journal in 1857, the speaker had given the differential equations of the catenary formed by a submarine cable between the ship and the bottom, during the submergence, under the influence of gravity and fluid friction and pressure; and he had pointed out that the curve becomes a straight line in the case of no tension at the bottom. As this is always the case in deep-sea cable laying, he made no farther reference to the general problem in the present address.

When a cable is laid at uniform speed, on a level bottom, quite straight, but without tension, it forms an inclined straight line, from the point where it enters the water, to the bottom, and each point of it clearly moves uniformly in a straight line towards the position on the bottom that it ultimately occupies.* That is to say, each particle of the cable moves uniformly along the base of an isosceles triangle, of which the two equal sides are the inclined portion of the cable between it and the bottom, and the line along the bottom which this portion of the cable covers when laid. When the cable is paid out from the ship at a rate exceeding that of the ship's progress, the velocity and direction of the motion of any particle of it through the water are to be found by compounding a velocity along the inclined side, equal to this excess, with the velocity already determined, along the base of the isosceles triangle.

The angle between the equal sides of the isosceles triangle, that is to say, the inclination which the cable takes in the water, is determined by the condition, that the transverse component of the cable's weight in water is equal to the transverse component of the resistance of the water to its motion. Its tension where it enters the water is equal to the longitudinal component of the weight (or, which is the same, the whole weight of a length of cable hanging vertically down to the bottom), diminished by the longitudinal component of the fluid resistance. In the laying of the Atlantic cable, when the depth was two miles, the rate of the ship six miles an hour, and the rate of paying out of the cable seven miles an hour, the resistance to the egress of the cable, accurately measured by a dynamometer, was only 14 cwt. But it must have been as much as 28 cwt., or the weight of two miles of the cable hanging vertically down in water, were it not for the frictional resistance of the water against the cable slipping, as it were, down an inclined plane from the ship to the bottom, which therefore must have borne the difference, or 14 cwt. Accurate observations are wanting as to the angle at which the cable entered the water; but from measurements of angles at the stern of the ship, and a dynamical estimate (from the measured strain) of what the curvature must have been between the ship and the water, I find that its inclination in the water, when the ship's speed was nearly $6\frac{1}{2}$ miles per hour, must have been about $6\frac{3}{4}°$, that is to say, the incline was about 1 in $8\frac{1}{2}$. Thus the length of cable, from the ship to the bottom, when the water was 2 miles deep, must have been about 17 miles.

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* Precisely the movement of a battalion in line changing front.

The whole amount (14 cwt.) of fluid resistance to the motion of this length of cable through it, is therefore about ·81 of a cwt. per mile. The longitudinal component velocity of the cable through the water, to which this resistance was due, may be taken, with but very small error, as simply the excess of the speed of paying out above the speed of the ship, or about 1 mile an hour. Hence, to haul up a piece of the cable vertically through the water, at the rate of 1 mile an hour, would require less than 1 cwt. for overcoming fluid friction, per mile length of the cable, over and above its weight in water. Thus fluid friction, which for the laying of a cable performs so valuable a part in easing the strain with which it is paid out, offers no serious obstruction, indeed, scarcely any sensible obstruction, to the reverse process of hauling back, if done at only 1 mile an hour, or any slower speed.

As to the transverse component of the fluid friction, it is to be remarked that, although not directly assisting to reduce the egress strain, it indirectly contributes to this result; for it is the transverse friction that causes the gentleness of the slope, giving the sufficient length of 17 miles of cable slipping down through the water, on which the longitudinal friction operates, to reduce the egress strain to the very safe limit found in the recent expedition. In estimating its amount, even if the slope were as much as 1 in 5, we should commit only an insignificant error, if we supposed it to be simply equal to the weight of the cable in water, or about 14 cwt. per mile for the 1865 Atlantic cable. The transverse component velocity to which this is due may be estimated with but insignificant error, by taking it as the velocity of a body moving directly to the bottom in the time occupied in laying a length of cable equal to the 17 miles of oblique line from the ship to the bottom. Therefore, it must have been about 2 miles in $17 \div 6\frac{1}{2} = 2·61$ hours, or ·8 of a mile per hour. It is not probable that the actual motion of the cable lengthwise through the water can affect this result much. Thus, the *velocity of settling* of a horizontal piece of the cable (or velocity of sinking through the water, with weight just borne by fluid friction) would appear to be about ·8 of a mile per hour. This may be contrasted with longitudinal friction by remembering that, according to the previous result, a longitudinal motion through the water at the rate of 1 mile per hour is resisted by only $\frac{1}{17}$th of the weight of the portion of cable so moving.

These conclusions justify remarkably the choice that was made of materials and dimensions for the 1865 cable. A more compact cable (one for instance with less gutta percha, less or no tow round the iron wires, and somewhat more iron), even if of equal strength and equal weight per mile in water, would have experienced less transverse resistance to motion through the water, and therefore would have run down a much steeper slope to the bottom. Thus, even with the same longitudinal friction per mile, it would have been less resisted on the shorter length; but even on the same length it would have experienced much less longitudinal friction, because of its smaller circumference. Also, it is important to remark that the roughness of the outer tow covering undoubtedly did very much to ease the egress strain, as it must have increased the fluid friction greatly beyond what would have acted on a smooth gutta percha surface, or even on the surface of smooth iron wires, presented by the more common form of submarine cables.

The speaker showed models illustrating the paying-out machines used on the Atlantic expeditions of 1858 and 1865. He stated that nothing could well be imagined more perfect than the action of the machine of 1865 in paying out the 1200 miles of cable then laid, and that if it were only to be used for *paying out*, no change either in general plan or in detail seemed desirable, except the substitution of a softer material for the "jockey pulleys," by which the cable in entering the machine has the small amount of resistance applied to it which it requires to keep it from slipping round the main drum. The rate of egress of the cable was kept always under perfect control by a weighted friction brake of Appold's construction (which had proved its good quality in the 1858 Atlantic expedition) applied to a second drum carried on the same shaft with the main drum. When the weights were removed from the brake (which could be done almost instantaneously by means of a simple mechanism), the resistance to the egress of the cable, produced by "jockey pulleys," and the friction at the bearings of the shaft carrying the main drum, &c., was about 2½ cwt.

Procedure to Repair the Cable in case of the appearance of an electric fault during the laying.

In the event of a fault being indicated by the electric test at any time during the paying out, the safe and proper course to be followed in future (as proved by the recent experience), if the cable is of the same construction as the present Atlantic cable, is instantly, on order given from an authorised officer in the electric room, to stop and reverse the ship's engines, and to put on the greatest *safe* weight on the paying-out break. Thus in the course of a very short time the egress of the cable may be stopped, and, if the weather is moderate, the ship may be kept, by proper use of paddles, screw, and rudder, nearly enough in the proper position for hours to allow the cable to hang down almost vertically, with little more strain than the weight of the length of it between the ship and the bottom.

The best electric testing that has been practised or even planned cannot show within a mile the position of a fault consisting of a slight loss of insulation, unless both ends of the cable are at hand. Whatever its character may be, unless the electric tests demonstrate its position to be remote from the outgoing part, the only thing that can be done to find whether it is just on board or just overboard, is to cut the cable as near the outgoing part as the mechanical circumstances allow to be safely done. The electric test immediately transferred to the fresh-cut seaward end shows instantly if the line is perfect between it and the shore. A few minutes more, and the electric tests applied to the *two ends* of the remainder on board, will, in skilful hands, with a proper plan of working, show very closely the position of the fault, *whatever its character may be.* The engineers will thus immediately be able to make proper arrangements for resplicing and paying out good cable, and for cutting out the fault from the bad part.

But if the fault is between the land end and the fresh-cut seaward end on board ship, proper simultaneous electric tests on board ship and on shore (not hitherto practised, but easy and sure if properly planned) must be used to discover whether the fault lies so near the ship that the right thing is to haul back the cable until it is got on board. If it is so, then steam power must be applied to reverse the paying-out machine, and, by careful watching of the dynamometer, and controlling the power accordingly (hauling in slowly, stopping, or veering out a little, but never letting the dynamometer go above 60 or 65 cwt.), the cable (which can bear 7 tons) will not break, and the fault will be got on board more surely, and possibly sooner, than a " sulky" salmon of 30 lbs. can be landed by an expert angler with a line and rod that could not bear 10 lbs. The speaker remarked that he was entitled to make such assertions with confidence now, because the experience of the late expedition had not only verified the estimates of the scientific committee and of the contractors as to the strength of the cable, its weight in water (whether deep or shallow), and its mechanical manageability, but it had proved that in moderate weather the Great Eastern could, by skilful seamanship, be kept in position and moved in the manner required. She had actually been so for thirty-eight hours, and eighteen hours during the operations involved in the hauling back and cutting out the first and second faults, and reuniting the cable, and during seven hours of hauling in, in the attempt to repair the third fault.

Should the simultaneous electric testing on board and on shore prove the fault to be 50 or 100 or more miles from the ship, it would depend on the character of the fault, the season of the year, and the means and appliances on board, whether it would be better to complete the line, and afterwards, if necessary, cut out the fault and repair, or to go back at once and cut out the fault before attempting to complete the line. Even the worst of these contingencies would not be fatal to the undertaking with such a cable as the present one. But all experience of cable-laying shows that almost certainly the fault would either be found on board, or but a very short distance overboard, and would be reached and cut out with scarcely any risk, if really prompt measures, as above described, are taken at the instant of the appearance of a fault, to stop as soon as possible with safety the further egress of the cable.

The most striking part of the Atlantic undertaking proposed for 1866, is that by which the 1200 miles of excellent cable laid in 1865 is to be utilised by completing the line to Newfoundland.

That a cable lying on the bottom in water two miles deep can be caught by a grapnel and raised several hundred fathoms above the bottom, was amply proved by the eight days' work which followed the breakage of the cable on the 3d of August last. Three times out of four that the grapnel was let down, it caught the cable, on each occasion after a few hours of dragging, and with only 300 or 400 fathoms more of rope than the 2100 required to reach the bottom by the shortest course. The time when the grapnel did not hook the cable it came up with one of its flukes caught round by its chain; and the grapnel, the short length of chain next it, and about 200 fathoms of the wire-rope, were proved to have been dragged along the bottom, by being found when brought on board to have interstices filled with soft light gray ooze (of which the speaker showed a specimen to the Royal Society). These results are quite in accordance with the dynamical theory indicated above (see Appendix II.), according to which a length of such rope as the electric cable, hanging down with no weight at its lower end, and held by a ship moving through the water at half a mile an hour, would slope down to the bottom at an angle from the vertical of only 22°; and the much heavier and denser wire-rope that was used for the grappling would go down at the same angle with a considerably more rapid motion of the ship, or at a much steeper slope with the same rate of motion of the ship.

The only remaining question is: How is the cable to be brought to the surface when hooked? The operations of last August failed from the available rope, tackle, and hauling machine not being strong enough for this very unexpected work. On no occasion was the electric cable broken.* With strong enough tackle, and a hauling machine, both strong enough, and under perfect control, the lifting of a submarine cable, as good in mechanical quality as the Atlantic cable of 1865, by a grapnel or grapnels, from the bottom at a depth of two miles, is certainly practicable. If one attempt fails, another will succeed; and there is every reason, from dynamics as well as from the 1865 experience, to believe that in any moderate weather the feat is to be accomplished with little delay, and with very few if any failing attempts.

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* The strongest rope available was a quantity of rope of iron wire and hemp spun together, able to bear 14 tons, which was prepared merely as *buoy-rope* (to provide for the contingency of being obliged, by stress of weather or other cause, to cut and leave the cable in deep or shallow water), and was accordingly all in 100 fathoms-lengths, joined by shackles with swivels. The

The several plans of proceeding that have been proposed are of two classes—those in which, by three or more ships, it is proposed to bring a point of the cable to the surface without breaking it at all; and those in which it is to be cut or broken, and a point of the cable somewhat eastward from the break is to be brought to the surface.

With reference to either class, it is to be remarked that, by lifting simultaneously by several grapnels so constructed as to hold the cable without slipping along it or cutting it, it is possible to bring a point of the cable to the surface without subjecting it to any strain amounting to the weight of a length of cable equal to the depth of the water. But so many simultaneous grapplings by ships crossing the line of cable at considerable distances from one another would be required, that this possibility is scarcely to be reckoned on practically, without cutting or breaking the cable at a point westward of the points raised by the grapnels. On the other hand, with but three ships the cable might, no doubt, be brought to the surface at any point along the line, without cutting it, and without subjecting it at any point to *much* more strain than the weight corresponding to the vertical depth, as is easily seen when it is considered that the cable was laid generally with from 10 to 15 per cent. of slack. And if the cable is cut at some point not far westward of the westernmost of the grapnels, there can be no doubt but it could be lifted with great ease by three grapnels hauled up simultaneously

wire and hemp rope itself never broke, but on two of the three occasions a swivel gave way. On the last occasion, about 900 fathoms of Manilla rope had to be used for the upper part, there not being enough of the wire buoy-rope left; and when 700 fathoms of it had been got in, it broke on board beside a shackle, and the remaining 200 fathoms of the Manilla, with 1540 fathoms of wire-rope and the grapnel, and the electric cable which it had hooked, were all lost for the year 1865.

by three ships. The catenaries concerned in these operations were illustrated by a chain with 15 per cent. of slack hauled up simultaneously at three points.

The plan which seemed to the speaker surest and simplest is to cut the cable at any chosen point, far enough eastward of the present broken end to be clear of entanglement of lost buoy-rope, grapnels, and the loose end of the electric cable itself; and then, or as soon as possible after, to grapple and lift at a point about three miles farther eastward. This could be well and safely done by two ships, one of them with a cutting grapnel, and the other (the Great Eastern herself) with a holding grapnel. The latter, on hooking, should haul up cautiously, never going beyond a safe strain, as shown by the dynamometer. The other, when assured that the Great Eastern has the cable, should haul up, at first cautiously, but ultimately, when the cable is got well off the bottom by the Great Eastern, the western ship should move slowly eastwards, and haul up with force enough to cut or break the cable. This leaves three miles of free cable on the western side of the Great Eastern's grapnel, which will yield freely eastwards (even if partly lying along the bottom at first), and allow the Great Eastern to haul up and work slowly eastwards, so as to keep its grappling rope, and therefore ultimately the portions of electric cable hanging down on the two sides of its grapnel, as nearly vertical as is necessary to make sure work of getting the cable on board. This plan was illustrated by lifting, by aid of two grapnels, a very fragile chain (a common brass chain in short lengths, joined by links of fine cotton thread) from the floor of the Royal Society. It was also pointed out that it can be executed by one ship alone, with only a little delay, but with scarcely any risk of failure. Thus, by first hooking the cable by a holding grapnel, and hauling it up 200 or 300 fathoms from the bottom, it may be left there hanging by the grapnel-rope on a buoy, while the ship proceeds three miles westwards, cuts the cable there, and returns to the buoy. Then, it is an easy matter, in any moderate weather, to haul up safely and get the cable on board.

The use of the dynamometer in dredging was explained; and the forces operating on the ship, the conditions of weather, and the means of keeping the ship in proper position during the process of slowly hauling in a cable, even if it were of strength quite insufficient to act, when nearly vertical, with any sensible force on the ship, were discussed at some length. The manageability of the Great Eastern, in skilful hands, had been proved to be very much better than could have been expected, and to be sufficient for the requirements in moderate weather. She has both screw and paddles—an advantage possessed by no other steamer in existence. By driving the screw at full power ahead, and backing the paddles, to prevent the ship from moving ahead, or (should the screw overpower the paddles), by driving the paddles full power astern, and driving at the same time the screw ahead with power enough to prevent the ship from going astern, "steerage way" is *created* by the lash of water from the screw against the rudder; and thus the Great Eastern may be effectually steered without going ahead. Thus she is, in calm or moderate weather, almost as manageable as a small tug steamer, with reversing paddles, or as a rowing boat. She can be made still more manageable than she proved to be in 1865, by arranging to disconnect either paddle at any moment; which, the speaker was informed by Mr Canning, may easily be done.

The speaker referred to a letter he had received from Mr Canning, chief engineer of the Telegraph Construction and Maintenance Company, informing him that it is intended to use three ships, and to be provided both with cutting and with holding grapnels, and expressing great confidence as to the success of the attempt. In this confidence the speaker believed every practical man who witnessed the Atlantic operations of 1865 shared, as did also, to his knowledge, other engineers who were not present on that expedition, but who were well acquainted with the practice of cable-laying and mending in various seas, especially in the Mediterranean. The more he thought of it himself, both from what he had witnessed on board the Great Eastern, and from attempts to estimate on dynamical principles the forces concerned, the more confident he felt that the contractors would succeed next summer in utilising the cable partly laid in 1865, and completing it into an electrically perfect telegraphic line between Valencia and Newfoundland.

APPENDIX I.

Descriptions of the Atlantic Cables of 1858 *and* 1865.

(Distance from Ireland to Newfoundland, 1670 Nautical Miles.)

Old Atlantic Cable, 1858.

Conductor.—A copper strand, consisting of seven wires (six laid round one), and weighing 107 lbs. per nautical mile.

Insulator.—Gutta percha laid on in three coverings, and weighing 261 lbs. per knot.

External Protection.—Eighteen strands of charcoal iron wire, each strand composed of seven wires (six laid round one), laid spirally round the core, which latter was previously padded with a serving of hemp saturated with a tar mixture. The separate wires were each 22 gauge; the stand complete was No. 14 gauge.

Circumference of Finished Cable, 2 inches.

Weight in Air, 20 cwt. per nautical mile.

Weight in Water, 13·4 cwt. per nautical mile.

Breaking Strain, 3 tons 5 cwt., or equal to 4·85 times the cable's weight in water per mile. Hence the cable would bear its own weight in nearly five miles depth of water, or 2·05 times the—

Deepest Water to be encountered, 2400 fathoms, being less than 2½ nautical miles.

Length of Cable Shipped, 2174 nautical miles.

New Atlantic Cable, 1865.

Conductor.—Copper strand consisting of seven wires (six laid round one), and weighing 300 lbs. per nautical mile, embedded for solidity in Chatterton's compound. Diameter of single wire ·048 = ordinary 18 gauge. Gauge of strand ·144 = ordinary No. 10 gauge.

Insulation.—Gutta percha, four layers of which are laid on alternately with four thin layers of Chatterton's compound. The weight of the entire insulation 400 lbs. per nautical mile. Diameter of core ·464 of an inch; circumference of core 1·46 inches.

External Protection.—Ten solid wires of diameter ·095 (No. 13 gauge) drawn from Webster and Horsfall's homogeneous iron, each wire surrounded separately with five strands of Manilla yarn, saturated with a preservative compound, and the whole laid spirally round the core, which latter is padded with ordinary hemp, saturated with preservative mixture.

Circumference of Finished Cable, 3·534 inches.

Weight in Air, 35 cwt. 3 qrs. per nautical mile.

Weight in Water, 14 cwt. per nautical mile.

Breaking Strain, 7 tons 15 cwt., or equal to eleven times the cable's weight in water per mile. Hence the cable will bear its own weight in eleven miles depth of water, or 4·64 times the—

Deepest Water to be encountered, 2400 fathoms, or less than $2\frac{1}{2}$ nautical miles.

Length of Cable Shipped, 2300 nautical miles.

II.

Let W be the weight of the cable per unit of its length in water; T the force with which the cable is held back at the point where it reaches the water (which may be practically regarded as equal to the force with which its egress from the ship is resisted by the paying-out machinery, the difference amounting only to the weight in air of a piece of cable equal in length to the height of the stern pulley above the water); P and Q the transverse and longitudinal components of the force of frictional resistance experienced by the cable in passing through the water from surface to bottom; i the inclination of its line to the horizon; D the depth of the water.

The whole length of cable from surface to bottom will be $\frac{D}{\sin i}$; and the transverse and longitudinal components of the weight of this portion are therefore $\frac{WD}{\sin i}\cos i$, and WD respectively. These are balanced by $P\frac{D}{\sin i}$ and $T + Q\frac{D}{\sin i}$.

Hence

$$P = W\cos i,\ Q = \left(W - \frac{T}{D}\right)\sin i \quad . \ . \ . \ . \ . \quad (1.)$$

To find the corresponding components of the velocity of the cable through the water, which we shall denote by p and q, we have only to remark that the actual velocity of any portion of the cable in the water may be regarded as the resultant of two velocities,—one equal and parallel to that of the ship forwards, and the other obliquely downwards along the line of the cable, equal to that of the paying out, obliquely downwards along the line of the cable (since if the cable were not paid out, but simply dragged, while by any means kept in a straight line at any constant inclination, its motion would be simply that of the ship). Hence, if v be the ship's velocity, and u the velocity at which the cable is paid out from the ship, we have

$$p = v \sin i,\ q = u - v \cos i \quad . \quad . \quad . \quad . \quad (2.)$$

Now, as probably an approximate, and therefore practically useful, hypothesis, we may suppose each component of fluid friction to depend solely on the corresponding component of the fluid velocity, and to be proportional to its square. Thus we may take

$$\mathrm{P} = \mathrm{W}\frac{p^2}{\mathfrak{p}^2}, \qquad \mathrm{Q} = \mathrm{W}\frac{q^2}{\mathfrak{q}^2} \quad . \quad . \quad . \quad . \quad . \quad (3.)$$

where $\mathfrak{p}$ and $\mathfrak{q}$ denote the velocities, transverse and longitudinal, which would give frictions amounting to the weight of the cable; or, as we may call them, the transverse and longitudinal *settling velocities*. We may use these equations merely as introducing a convenient piece of notation for the components of fluid friction, without assuming any hypothesis, if we regard $\mathfrak{p}$ and $\mathfrak{q}$ as each some unknown function of p and q. It is probable that $\mathfrak{p}$ depends to some degree on q, although chiefly on p; and *vice versa*, $\mathfrak{q}$ to some degree on p, but chiefly on q. It is almost certain, however, from experiments such as those described in "Beaufoy's Nautical Experiments," that $\mathfrak{p}$ and $\mathfrak{q}$ are each *very nearly* constant for all practical velocities.

Eliminating p and q between (1), (2), and (3), we have

$$\mathrm{W} \cos i = \mathrm{W}\left(\frac{v \sin i}{\mathfrak{p}}\right)^2,$$

which gives

$$\mathfrak{p} = \frac{v \sin i}{\sqrt{\cos i}} \quad . \quad . \quad . \quad . \quad (4.)$$

and

$$(\mathrm{WD} - \mathrm{T}) \sin i = \mathrm{WD}\left(\frac{u - v \cos i}{\mathfrak{q}}\right)^2 \quad . \quad . \quad . \quad . \quad (5.)$$

which gives

$$\mathfrak{q} = (u - v \cos i)\sqrt{\frac{\mathrm{WD}}{(\mathrm{WD} - \mathrm{T}) \sin i}} \quad . \quad . \quad . \quad . \quad (6.)$$

These formulæ apply to every case of uniform towing of a rope under water, or hauling in, or paying out, whether the lower end reaches the bottom or not, provided always the lower end is free from tension; but if it is not on the bottom, D must must denote its vertical depth at any moment, instead of the whole depth of the sea. To apply to the case of merely towing, we must put $u=0$; or, to apply to hauling in, we must suppose u negative.

It is to be remarked that the inclination assumed by the cable under water does not depend on its longitudinal slip through the water (since we assume this not to influence the transverse component of fluid friction), and that, according to equation (4), it is simply determined by the ratio of the ship's speed to the transverse "settling velocity" of the cable.

The following table shows the ratio of the ship's speed to the "transverse settling velocity" of the cable for various degrees of inclination of the cable to the horizon :—

	Inclination of Cable to Horizon.	Ratio of Ship's Speed to "*transverse settling velocity*" of Cable.	Inclination of Cable to Horizon.	Ratio of Ship's Speed to "*transverse settling velocity*" of Cable.
	i	$\frac{v}{\mathfrak{p}}=\frac{\sqrt{\cos i}}{\sin i}$	i	$\frac{v}{\mathfrak{p}}=\frac{\sqrt{\cos i}}{\sin i}$
	5°	11·4518	45°	1·1892
angle whose sine is $\frac{1}{8\frac{1}{2}}$	6° 45′	8·4784	50	1·0466
	10	5·7149	51° 50′	1·0000
	15	3·7973	55	·9232
	20	2·8343	60	·8165
	25	2·2013	65	·7173
	30	1·8612	70	·6224
	35	1·5779	75	·5267
	40	1·3616	80	·4231
			85	·0875

If the inclination of the cable had been exactly 6° 45′ when the speed of the Great Eastern was exactly 6½ miles per hour, the value of $\mathfrak{p}$ for the Atlantic cable of 1865 would be exactly $6\frac{1}{2}\div 8{\cdot}478$, or ·765 of a mile per hour.

The following Communications were read :—

1. On the Dynamical Theory of Heat. Part VII. By Professor W. Thomson.

This paper commences with a condensed re-statement of the fundamental principles and formulæ of the Dynamical Theory of Heat, from the first six parts of the author's treatment of the subject previously communicated to the Royal Society of Edinburgh, and his articles "On the Thermo-elastic Properties of Matter," in the "Quarterly Mathematical Journal" (April 1855), and on "Thermo-magnetism," and "Thermo-electricity," in Nichol's Cyclopedia (Edinburgh 1860).

The chief object of the paper is the deduction of numerical values in absolute measure for the thermo-electric effects which form the subject of Part VI. of this series ("Transactions of the Royal Society of Edinburgh," 1854; and "Phil. Mag." 1854, second half year, and 1855, first half year), especially for differences of temperature produced by electric convection of heat, and for the changes of temperature due to strain in elastic solids, investigated in the article on thermo-elastic properties of matter above referred to. The very valuable results, recently published, of the experiments of Forbes and Ångström for determining in absolute measure the thermal conductivities of iron and copper, supply a very important element, previously wanting, for definite estimates of those changes of temperature, and are taken advantage of in the present paper. Thus, the author has been enabled to give that practical character to some of his former conclusions, of which, when they were first published, he pointed out the want. In particular, with reference to elastic solids, the *apparent* value of Young's modulus * when the stress is applied and removed, or reversed so rapidly that the loss of thermal effect by conduction and radiation is insensible, is proved to be given by the following formula :—

$$M' = M\left(1 + \frac{e^2 t M}{J s \rho}\right)$$

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* The amount of the force divided by the elongation produced by it, when any force within practical limits of elasticity is applied to elongate a bar rod or wire, of the substance, one square centimetre in section.

where M denotes the Young's modulus of the substance for constant temperature, s its specific heat (per unit mass, as usual), e its longitudinal (linear) expansion per degree of elevation of temperature, ρ its density or specific gravity,* and t its actual temperature from absolute zero ("Dynamical Theory of Heat," Part VI., § 100), that is, temperature centigrade with 274 added. Of course, if M is reckoned ("Thomson and Tait's Natural Philosophy," §§ 220, 221, 238), in gravitation measure (weight of one gramme, the unit of mass), J must be reckoned in gravitation measure (grammes weight working through one centimetre), in which case its numerical value is 42,400, being Joule's number (1390), reduced from feet to centimetres. Values of surface resistance to gain or loss of heat in absolute measure, derived from experiments by the author, are used to estimate the effect of radiation and convection in dissipating energy in virtue of the thermo-dynamic change of temperature in a rod executing longitudinal vibrations. The velocity of propagation of longitudinal vibrations (as in the transmission of sound along a bar) being equal to the velocity acquired by a body in falling through a height equal to half the "length of the modulus,"† is, of course, half as much affected as the modulus, by changes of temperature. In iron, for instance, the effect of change of temperature, when there is no dissipation, is an increase of about one-third per cent. on the Young's modulus, and of about one-sixth per cent. on the velocity of sound along a bar. The effect of the conduction of heat in diminishing the differences of temperature in a rectangular bar executing flexural vibrations, is investigated from the solution invented by Fourier for expressing periodical variations of underground temperature. Its absolute amount for bars of iron or copper, of stated dimensions, vibrating in stated periods, is determined from Forbes' and Ångström's conductivities. It is proved that the loss of energy due to this effect at its maximum is not by any means insensible, though it is not sufficient to account for the whole loss of energy which the author has found in experiments on flexural vibrations of metal springs, which therefore prove imperfectness in the elasticity of flexure, such as he had previously proved for the elasticity of torsion.*

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* Which, when the French system (unit bulk of water being of mass unity) is followed, mean the same thing

† The "length of the modulus" is $M \div \rho$, if M be the modulus in grammes weight per square centimetre.—*Thomson and Tait's Natural Philosophy*, § 689.

2. The "Doctrine of Uniformity" in Geology briefly refuted. By Professor William Thomson.

The "Doctrine of Uniformity" in Geology, as held by many of the most eminent of British Geologists, assumes that the earth's surface and upper crust have been nearly as they are at present in temperature, and other physical qualities, during millions of millions of years. But the heat which we know, by observation, to be now conducted out of the earth yearly is so great, that if *this* action had been going on with any approach to uniformity for 20,000 million years, the amount of heat lost out of the earth would have been about as much as would heat, by 100° Cent., a quantity of ordinary surface rock of 100 times the earth's bulk. (See calculation appended.) This would be more than enough to melt a mass of surface rock equal in bulk to the *whole earth.* No hypothesis as to chemical action, internal fluidity, effects of pressure at great depth, or possible character of substances in the interior of the earth, possessing the smallest vestige of probability, can justify the supposition that the earth's upper crust has remained nearly as it is, while from the whole, or from any part, of the earth, so great a quantity of heat has been lost.

APPENDIX.

Estimate of present annual loss of heat from the earth.

Let A be the area of the earth's surface, D the increase of depth in any locality for which the temperature increases by 1° Cent., and k the conductivity per annum of the strata in the same locality. The heat conducted out per annum per square foot of surface in that locality is $\frac{k}{D}$. Hence, if we give k and D proper average values for the whole upper crust of the earth, the quantity conducted out across the whole earth's surface per annum will be $\frac{kA}{D}$. The bulk of a sphere being its surface multiplied by $\frac{1}{3}$ of its radius, the thermal capacity of a mass of rock equal in bulk to the earth, and of specific heat s per unit of bulk is $\frac{1}{3}Ars$. Hence $\frac{3k}{Drs}$ is the elevation of temperature which a quantity of heat equal to that lost from the earth in a year, would produce in a mass of rock equal in bulk to the whole earth. The laboratory experiments of Peclet; Observations on Underground Temperature in three kinds of rock in and near Edinburgh, by Forbes; in two Swedish strata, by Ångström, and at the Royal Observatory, Greenwich, give values of the conductivity in gramme-water units of heat per square centimetre, per 1° per centimetre of variation of temperature, per second, from ·002 (marble, Peclet) to ·0107 (sandstone of Craigleith quarry, Forbes); and ·005 may be taken as a rough average. Hence, as there are 31,537,600 seconds in a year, we have $k = \cdot 005 \times 31{,}537{,}600$, or approximately 15×10^4. The thermal capacity of surface rock is somewhere about half that of equal bulk of water; so that we may take $s = \cdot 5$. And the increase of temperature downwards may be taken as roughly averaging 1° Cent. per 30 metres; so, that, $D = 3000$ centimetres. Lastly, the earth's quadrant being according to the first foundation of the French metrical system, about 10^9 centimetres, we may take, in a rough estimate such as the present, $r = 6 \times 10^8$ centimetres. Hence,

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* Proceedings of the Royal Society of London, May 1865.—*W Thomson, "On the Elasticity and Viscosity of Metals."*

$$\frac{3k}{Drs} = \frac{3 \times 15 \times 10^4}{3000 \times 6 \times 10^8 \times \cdot 5} = \frac{30 \times 10^4}{10^{11} \times 6} = \frac{5}{10^7}$$

This, multiplied by $20{,}000 \times 10^6$, amounts to 10,000, or to 100 times as much heat as would warm 100 times the earth's bulk of surface rock by 1° Cent.

3. Note on the Atomicity of Sulphur. By Dr Alexander Crum Brown.

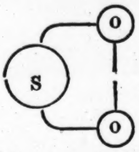
We now know a considerable number of elements which exhibit, in their compounds, two or more distinct degrees of atomicity. Thus we have N triatomic in ammonia and its analogues, in nitrous acid, the nitrites and the nitroso-substitution products; pentatomic in the ammonia salts, in nitric acid, and the nitro-substitution products. P triatomic in PH_3, PCl_3, phosphorous acid, and the phosphites; pentatomic in PCl_5, $POCl_3$, H_3PO_4, HPO_3, &c.; C diatomic in CO, and tetratomic in almost all other compounds; a large number of such examples might be adduced, but these may suffice.

No chemist, however, as far as I know, except Kolbe and v. Oefele has assumed a similar variety in the atomicity of sulphur. I had long suspected that such a variable atomicity must be attributed to sulphur, but the remarkable discoveries of v. Oefele have to my mind placed it beyond a doubt. Indeed, so strong does the evidence appear to me, that I should not have thought it necessary to trouble the Society with this note, had I not observed that some of our most eminent theoretical chemists, as Hofmann and Foster, still represent S as diatomic in such compounds as SO_2 and $H_2\,SO_4$. I have therefore gathered together what seem the most cogent arguments in favour of the variable atomicity of sulphur.

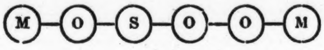
That S is diatomic in H_2S and the corresponding metallic sulphides, in the mercaptans and alkyl-sulphides, is quite obvious, and is admitted by all. I need therefore spend no time on this part of the subject. Let us then turn to those compounds in which there is reason to suspect the presence of tetratomic sulphur. These are SO_2, $SO\,Cl_2$, M_2SO_3, $S(C_2H_5)_3I$, and its derivatives. The latter series of substances discovered by v. Oefele seem to me quite inexplicable, except by the assumption of tetratomic sulphur, and I have never seen an attempt to explain their constitution in any other way.* But even leaving them out of the question, the most natural explanation of the constitution of SO_2 and M_2SO_3 is, that their sulphur atom is tetratomic. Formulæ can, indeed, be constructed with diatomic sulphur, which represent the *composition* of these bodies; but if we examine these formulæ we easily see that

* Wurtz, in a note to a paper by v. Oefele on "Diethyl Sulphan" in the Bulletin de la Société Chimique de Paris for February 1865, formulates that among other sulphur compounds on the assumption of diatomic sulphur. But it must be remembered that this body $(C_2H_5)_2SO_2$, contains in the molecule only two monad radicals, so that it takes its place beside chloride of sulphuryl rather than in the series, since discovered by v. Oefele, of which the starting-point is the iodide of triethyl sulphin $S(C_2H_5)_3I$.

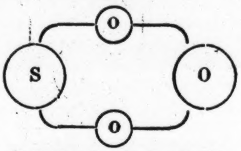
they are in many points of view forced and unnatural; in other words, that they are not what an unbiased consideration of the composition and properties of the substances would naturally lead to. This is still more the case with those substances in which the sulphur appears to be hexatomic. These are SO_3, M_2SO_4, $M_2S_2O_6$, and the numerous derivatives of H_2SO_4, which contain $(SO_2)''$; as these bodies, like the sulphites, contain, in the molecule, only two monad atoms, it is, of course, *possible* to represent their composition by formulæ containing diatomic sulphur.* Thus—



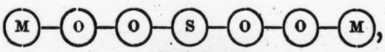
Sulphurous Anhydride.



Sulphite.



Sulphuric Anhydride.



, &c.

Sulphate.

Examining these formulæ we find that these stable substances have some of the oxygen equivalents saturated by oxygen; this is, of course, possible; indeed we know bodies such as H_2O_2, BaO_2, Ag_2O_2, Brodie's peroxides, &c., in which this arrangement is probable; but all these bodies are remarkable for instability, and for the readiness with which they lose oxygen. They present absolutely no points of resemblance to the substances we are now considering. The striking analogy of SO_2 and SO_3 to CO and CO_2 also leads to the conclusion that the former is $S^{iv}O_2$ and the latter $S^{vi}O_3$, and that in both of them all the oxygen is combined with sulphur. If we accept this view, and follow out the analogy between the carbon and the sulphur compounds, we get a simple explanation of many relations otherwise difficult to be understood. For instance, hypo-sulphuric acid obviously stands in the same relation to sulphurous anhydride and sulphuric acid as oxalic acid does to carbonic oxide and carbonic acid; and the manner in which hypo-sulphuric acid is formed by the reaction of sulphurous acid on peroxide of manganese further confirms this view of its constitution. This reaction may be represented thus—

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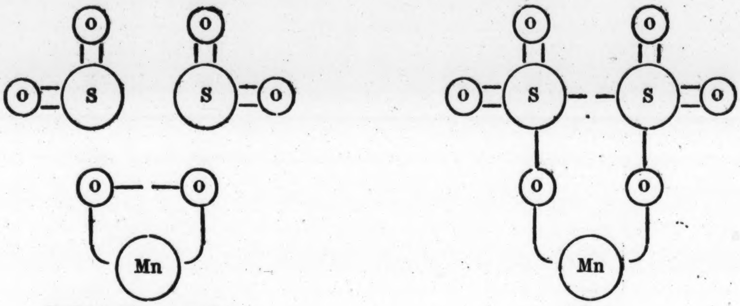
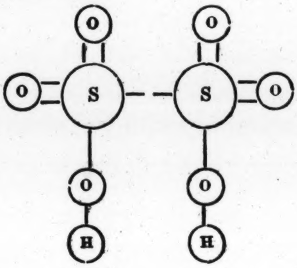
* I use here, for the purpose of representing the chemical structure of molecules, the graphic method which I first introduced in my thesis presented to the Medical Faculty of the University in 1861, and which I have more than once employed in papers read before this Society. I am happy to observe that its advantages are appreciated by others, Dr Hofmann, for instance, using a system almost identical in his lecture delivered before the Royal Institution on the 7th of April of this year.

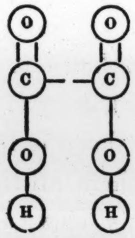


Before the reaction. After.

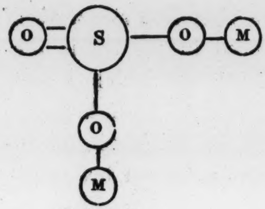
so that hypo-sulphuric acid is



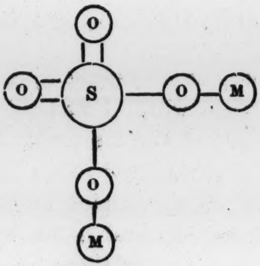
just as oxalic acid is



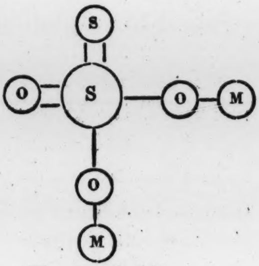
By means of the same assumption we get a simple explanation of the relations of the sulphites to sulphates, and to hypo-sulphites. Thus—



Sulphite.



Sulphate.



Hypo-sulphite.

The two latter bodies being respectively the oxide and the sulphide of the former, as their formation and decomposition indicate. It would be easy to give a much larger number of examples, but these are sufficient to show the way in which this view of the atomicity of sulphur may be applied to explain the constitution of its compounds.

4. Note on a paper by Balfour Stewart, Esq., in the Transactions of the Royal Society of Edinburgh, by I. Todhunter, Esq., M.A., St John's College, Cambridge. Communicated by Professor Tait.

In Volume XXI. of the Transactions of the Royal Society of Edinburgh, pages 407–409, a proposition in the Theory of Numbers is demonstrated. The proposition may be extended, and, at the same time, the demonstration simplified. I propose to establish the following result:—Let 1, α, β, γ, . . . μ, be the n roots of the equation

$$x^n - 1 = 0 \quad . \quad . \quad . \quad . \quad . \quad (1.)$$

then, will

$$(1-t)(1-t\alpha)(1-t\beta)(1-t\gamma) \quad . \quad . \quad . \quad (1-t\mu) = 1 - t^n \qquad (2.)$$

where t is any quantity.

It is obvious that the term independent of t on the left-hand side of (2) is unity. Consider any other term,—for example, that involving t^3; the coefficient of this is equal to the sum of the products of every three of the quantities 1, α, β, γ, . . . μ, with the sign changed; and from (1) we know that this sum is zero. In this way (2) is established.

It is known that if n be a prime number, the quantities α, β, γ, . . . μ, may be expressed as the powers of *any* one of them; for example, as α, α^2, α^3 . . α^{n-1}. And if n be not a prime number, they may be expressed as powers of *some* of them.

Divide both sides of (2.) by $1-t$; thus—

$$(1-\alpha t)(1-\beta t)(1-\gamma t) \quad . \quad . \quad . \quad (1-\mu t) = 1 + t + t^2 + \quad . \quad . \quad . \quad + t^{n-1}.$$

In this result, suppose $t=1$; then we obtain

$$(1-\alpha)(1-\beta)(1-\gamma) \quad . \quad . \quad (1-\mu) = n.$$

This is the Proposition established in Volume XXI.

[Mr Todhunter's elegant demonstration may be farther simplified, thus,—we have for all values of x,

$$x^n - 1 = (x-1)(x-\alpha)(x-\beta) \;\; . \;\; . \;\; . \;\; (x-\mu).$$

From this

$$x^{n-1} + x^{n-2} + \;\; . \;\; . \;\; . + x + 1 = (x-\alpha)(x-\beta) \;\; . \;\; . \;\; . \;\; (x-\mu)$$

or, putting $x = 1$

$$n = (1-\alpha)(1-\beta) \;\; . \;\; . \;\; . \;\; (1-\mu).$$

P. G. T.]

At the Meeting on 4th December, the following Gentlemen were elected Honorary Fellows of the Society:—

I. FOREIGN.

Angelo Secchi, *Observatory, Rome.*

II. BRITISH.

Lieut.-General **Edward Sabine**, R.A., *President of the Royal Society of London.*
Charles Darwin, Esq., M.A., *Down, Bromley, Kent.*
Arthur Cayley, Esq., *Professor of Mathematics, Cambridge.*

The following Gentlemen were elected Ordinary Fellows at the Meeting of 18th December:—

The Right Rev. **Bishop Morrell.**
William Euing, Esq., Glasgow.

The following Donations to the Library were announced:—

Proceedings and Transactions of the Nova Scotia Institute of Natural Science. Vol. II. Part 3. Halifax, 1865. 8vo.—*From the Institute.*

Medico-Chirurgical Transactions. Vol. XLVIII. London, 1865. 8vo.—*From the Royal Medico-Chirurgical Society.*

Quarterly Journal of the Geological Society. Vol. XXI. Part 4. London, 1865. 8vo.—*From the Society.*

Proceedings of the Royal Society, London. Vol. XIV. No. 78. 8vo.—*From the Society.*

The Canadian Journal of Industry, Science, and Art. No. 49. Toronto, 1865. 8vo.—*From the Editors.*

Journal of the Chemical Society, London. No. 35. 8vo.—*From the Society.*

Journal of the Proceedings of the Linnean Society (Zoology). Vol. VIII. Nos. 31–32. London, 1865. 8vo.—*From the Society.*

Memoirs of the Royal Astronomical Society. Vol. XXXIII. London, 1865. 4to.—*From the Society.*

Researches on Solar Physics. By Warren de la Rue, Esq., Balfour Stewart, Esq., and Benjamin Loewy, Esq. London, 1865. 4to. —*From the Authors.*

Thirty-Eighth Annual Report of the Council of the Royal Scottish Academy. Edinburgh, 1865. 8vo.—*From the Academy.*

Monthly Return of the Births, Deaths, and Marriages registered in the Eight Principal Towns of Scotland, November 1865. 8vo. —*From the Registrar-General.*

Mémoires de l'Académie Impériale des Sciences de St. Petersbourg. VII^e Série. Tome VII., Tome VIII. 4to.—*From the Academy.*

Bulletin de l'Académie Impériale des Sciences, de St Petersbourg. Tome VII. Nos. 3–6; Tome VIII. 4to.—*From the Academy.*

Norges Mynter i Middelalderen, samledeog beskrevne af C. I. Schive. Sjette Hefte, Femte Hefte. Christiania, 1865. Fol. —*From the Royal University of Norway.*

Nyt Magazin for Naturvidenskaberne. Trettende Binds, Fjerde Hefte; Fjortende Binds, Forste Hefte. Christiania, 1864–65. 8vo.—*From the Royal University of Norway.*

Norges Ferskvandskrebsdyr Forsted afsnit Branchiopoda, i Cladocera otenopoda, af Georg Ossian Sars. Christiania, 1865. 4to.—*From the Royal University of Norway.*

Om de i Norge Forskommende Fossile Dyrelevninger fra Quartær perioden, et Bidrag til vor Faunas Historie, af Dr Sars. Christiania, 1865. 4to.—*From the Royal University of Norway.*

Meteorologische Beobachtungen; aufgezeichnet auf Christiania Observatorium. Band I. 1837–63. Christiania, 1865. 4to. —*From the Royal University of Norway.*

Meteorologiske Iagttagelser paa, Christiania Observatorium, 1864. 4to.—*From the Royal University of Norway.*

Flateyjarbok en samling af Norske Konge-sagaer med imdskudte mindre fortallinger om begivenheder i og udenfor Norge Samt

Annaler, III. Binds, 1 Hefte. Christiania, 1865. 8vo.—*From the Royal University of Norway.*

Det Kongelige Norske Videnskabers-selskabs Skriften det 19de Aarhundrede, V. Binds, 1 Hefte. Throndhjem, 1865. 8vo.—*From the Royal University of Norway.*

Gaver til det Kgl. Norske Universitets i Christiania. 8vo.—*From the Royal University of Norway.*

Det Kongl. Norste Frederiks Universitets Aarsberetning for Aaret 1863. Christiania, 1865. 8vo.—*From the Royal University of Norway.*

Norske Universtets og Skole-Annaler udgivne af Universitetets Secretair; Mai, Oct. 1859; Marts, Juni 1860; Marts 1861; Marts 1862; Marts, Decr. 1863; Juni, Oct. 1864; Febr., Mai 1865. Christiania. 8vo.—*From the Royal University of Norway.*

Bulletin de l'Académie Royale des Sciences, des Lettres, et des Beaux-Arts de Belgique. Tome XX. Nos. 9, 10. Bruxelles, 1865. 8vo.—*From the Academy.*

The Lord Provost's Statement to the Town-Council respecting Sanitary Improvement. Edinburgh, 1865. 8vo.—*From the Right Hon. The Lord Provost of Edinburgh.*

Tuesday, 2d *January* 1866.

HON. LORD NEAVES, Vice-President, in the Chair.

The following Communications were read:—

1. Additional Observations on the Polarisation of the Atmosphere made at St Andrews in 1841, 1842, 1843, 1844, and 1845. By Sir David Brewster, K.H., D.C.L., F.R.S., &c.

In a former paper on this subject,* the author gave a selection from his observations on the polarisation of the atmosphere. After its publication, he received a long and elaborate memoir on the same subject by Dr R. Rubenson,* containing a series of valuable observations made at Rome on the place of maximum polarisation, and on the intensity of the maximum polarisation at different hours of the day. Important observations on the polarisation of the atmosphere have also been made by M. Liais and M. Andres Poey, and the value of such observations, in determining the height and constitution of our atmosphere, has been universally recognised.

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* Transactions, vol. xxiii. p. 211.

Under this impression, the author was induced to submit to the Society the rest of the four years' observations which he made at St Andrews, which, along with those already published, all exhibit the optical condition of the atmosphere during many days of every month of the year.

2. Notices of some Ancient Sculptures on the Walls of Caves in Fife. By Professor J. Y. Simpson.

The county of Fife abounds in caves or "weems"—a derivative from the Gaelic name for caves—and their existence gives a title to the earldom of Wemyss. Some of the caves in Fife are historical, as St Rule's at St Andrews, St Adrian's near Elie, and St Margaret's at Dunfermline. St Serf of Culross, the great patron saint of the west of Fife, is described by one of his biographers as having usually spent the forty days of Lent in a cave named, as such retreats often were, the *Desertum*. This cave at the *Desertum*—(or Dysart, to use the modern form of the name)—was used as a church up till near the time of the Reformation. About two miles eastward of Dysart, and near the village of Easter Wemyss, there is a range of large caves, seven or eight of which are at the present time open; but several more probably exist, having their openings covered over with debris. They stand about 15 or 20 feet above the level of high tide. Some of them are 80 to 100 feet in length, and of corresponding height and breadth. Two or three of them are perfectly dark, and require to be entered with candles. Last summer, when on a professional visit to Fife, Dr Simpson made a hurried visit to two of these caves, along with Dr Dewar, and saw some rude sculpturings in one of them. This discovery induced him to return for further search, accompanied by his friends, Drs Joseph Robertson, Duns, and Paterson, when two or three new caves were visited, and their walls found to be covered at different points with representations of various animals, figures, and emblems.

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* Acts of the Royal Society of Sciences of Upsal. Series iii. tom. v.

The cave sculpturings in Fife are of special interest to the Scotch archæologist, for this reason, that they exactly resemble, in type and character, the carvings on the so-called Sculptured Stones of Scotland. In his magnificent first volume on the Sculptured Stones of Scotland, Mr Stuart has collected one hundred and fifty examples; and latterly perhaps fifty more have been discovered. These Sculptured Stones extend along the whole east coast of Scotland, from the Forth northwards. Only two have been found south of the Forth. In general ornamentation, they resemble the sculptured stones of the west of Scotland, Ireland, Wales, and England; but the peculiarity of the Scotch stones is, that they have additional figures and symbols upon them that have been seen nowhere else in the world. These peculiar and characteristic symbols consist of the crescent or crescent-ornament, sometimes intersected with the V sceptre; of the so-called spectacle ornament—a double set of circles connected by middle lines—with or without the Z sceptre; of figures of elephants, fish, serpents, mirrors, combs, arches, or torcs, &c. The arrangement of these symbols upon the stones is in no two instances alike. On the oldest stones they are cut upon unhewn blocks, without any surrounding ornamentation. In the Sculptured Stones of a later date, they are cut in a raised form, with surrounding ornamentations, and often combined with figures of the Christian cross. Other figures are found carved on these stones, as portraits of priests and dignitaries, processions of men; the sacrifice of the bull; war and hunting scenes; animals, native and foreign, as the lion, tiger, camel, and monkey; the battling and devouring of men by wild animals; men with monster heads of beasts and birds; representations of dragons and monsters, &c. There is one instance of the representation of a boat and another of a chariot, at Meigle.

These rude sculpturings have, with one exception, been previously to the present time found only on sepulchral stones; but in the Fife caves they exist in great abundance on the cave walls.

These walls are usually comparatively smooth; and in many places, though not in all, they retain the figures cut upon them. The cave figures consist of animals, as the elephant,—exactly of the form seen on the Sculptured Stones,—the deer, the dog, the swan, the peacock, fish, serpents, and monsters. On them we see also representations of the mirror, comb, and arch or horse-shoe. No perfect example of the crescent ornamentation exists in these cave sculpturings; but many specimens of the spectacle ornament are to be found on their walls, both with and without the intersecting Z sceptre.

One of the cave-figures is specially interesting, from the fact that it is the exact counterpart of the only analogous carving found on aught except a monolith, viz., a scale of silver armour presented to the Antiquarian Museum of Scotland by Mrs Durham of Largo, and whose history is this:—A man still living in Fife—a huckster—acting, it is said, upon an old tradition, that a knight lay buried in silver armour in a small barrow called Norrie's Law, stealthily dug into it, found in reality the silver armour, and removed and sold it in pieces to the amount, it is alleged, of four hundred ounces. By the time this spoliation was discovered, the silver armour was all melted, except a few fragments. One of these fragments is a scale, having cut upon it a spectacle ornament traversed by the Z sceptre, and having appended to one end of it the head and shoulders of a dog, as in some modern Orders of European knighthood. Precisely a similar figure, with the appended dog's head, is carved upon the interior of one of the Wemyss caves.

On the walls of some of the caves there are crosses of various forms; and in two or three parts appearances somewhat resembling letterings, and symbolic arrangements of figures or hieroglyphics. On the walls of St Adrian's caves are lines which some have believed to be half obliterated Runes; and the Rev. Mr Skinner of St Andrews has a loose stone from this spot which presents, he thinks, Runic characters.

Among the cave sculpturings at Wemyss, there is a figure of a man of diminutive form; and Mr Stuart has traced among them faded outlines of a full sized human figure, apparently tailed, as if he formed one of the provokingly missing links which some enthusiastic ethnologists are so anxiously and vainly searching after.

The caves of Fife, both those that have sculptures and those without them, have almost all occasional complete perforations or holes cut in their sides, and in their floors and roof, capable of allowing a thong or rope to be passed through, as if they were intended to suspend or to affix objects.

The age of these cave-sculptures can only be fixed by approaching the age of the analogous figures upon the Sculptured Stones. The earliest of the Sculptured Stones are perhaps very old—possibly as far back, if not farther, than the period of the Roman invasion. In opening last year a large cairn at Linlethan in Forfarshire, a figure of the elephant, exactly similar to those existing on our sculptured stones, was found on a stone lying upon the covering of the stone-enclosed cist. This cist contained a bronze weapon and an urn. The elephant sculpture was as old, therefore, as the era of urn burial and bronze weapons—except the carved fragment of stone had got by pure accident into its present position when the barrow was opened twenty years ago. The ancients sometimes buried both stone and bronze relics with their dead, after apparently they had iron instruments and weapons. But if the bronze dagger at Linlethan was a weapon used by the person buried under the cairn, the date is probably pre-Roman. For when Agricola invaded Scotland in A.D. 81, our Caledonian forefathers had apparently already passed through the bronze era, as, according to Tacitus, they fought the Roman legions with swords "long and without a point;" in other words, with iron swords.

But most of the Sculptured Stones, particularly the more elaborate varieties of them, were of comparatively later date, and were probably erected as late as the eighth or tenth century. An elaborate specimen found buried in the old churchyard of St Vigeans, having upon its surface the spectacle ornament, the crescent, the mirror, the comb, &c., in raised figures, has an inscription on it, which is probably the only Pictish inscription and sentence now remaining. It speaks of the stone as erected to Drosten, the son of Voret; and a Pictish king Drosten was killed in the battle of Blathmig or Blethmont—a mile or two off—in the year 729, as we learn from the Annals of Tighearnach. The Fife cave sculptures at Wemyss are mixed up with numerous forms of crosses, particularly of equal-limbed Greek crosses, showing that they were cut after the introduction of Christianity; and in one or two spots there are appearances of Christian monograms. Within St Adrian's cave at Caplawchy, near Elie, there are many crosses on the walls; stone seats cut out, &c.; but no animals or symbols.

The purposes for which they were cut, and the meaning of the mysterious symbols on the caves and sculptured stones, are archæological enigmata that no one has yet solved. As long as they were found on sepulchral monoliths only, they were supposed to be hieroglyphic or heraldic *funeral* inscriptions or emblems. This doctrine is so far gainsaid by this late discovery of them on the walls of caves. But possibly they may be sacred symbols of some description, or of some unknown form and meaning. For around and upon his gravestones man has always been in the habit of cutting emblems of his religion when he has cut anything; things sacred and things sepulchral being found united in his earliest carvings.

Other Scotch caves have sculptures cut upon them. The so-called Cave of Bruce, in the Island of Arran, has been found by Dr Mitchell and Mr Stuart to have deer and serpents carved on its walls; and many years ago, within St Maloe's cave in Holy Island, Dr Daniel Wilson found ancient Scandinavian inscriptions written in Runes.

In many counties in Scotland, both on the sea shore and inland, there exist large caves, the walls of which require to be now carefully examined, in order to find if our ancient forefathers had carved upon them any such emblems and sculpturings as have been traced in Fife. The Fife caves have formerly been inhabited. Dr Simpson showed from one of the Wemyss caves a collection of bones which had been split to remove their marrow, like the bones found in the old Danish midden heaps, &c. Among the bones were those of the deer, sheep, ox, &c. There were also shells of limpets, &c.; and microscopic remains of cereals were found in cavities in the rocks that had been apparently used as rubbers or querns. Perforated stones and two implements from the tyre of the deer's horn were picked up from the rubbish upon the floor; but the debris of these caves requires to be most carefully searched, before all that could be ascertained on this point becomes known to archæologists. In Scotland, there is one cave still occasionally inhabited, at Wick, and within which Dr Mitchell has seen living a family of eight or ten. But cave men are common elsewhere. Mr Barnwell has lately recorded the curious fact, that in the neighbourhood of Charters there are at present living, in caves, 150,000 men, in the very centre of France. In Africa, Asia, &c., caves are still inhabited, as they were by the Troglodites and Horites of old.

In England, we know that in archaic times caves were inhabited by the men of those distant ages, such as Kent's Hole, the Brixham Cave, the Kirkdale caves, &c. In these caves the bones of man have been found with his stone weapons, and along with them the bones of long extinct animals, as the mammoth, the cave bear, the hyæna, &c. But in his earliest and rudest times, man has been a sculpturing and painting animal; and his old attempts in this way may yet be found upon the walls of those ossiferous English caves. Sir Charles Nicolson had stated to Dr Simpson, as a proof that man, in his savage state, was a sculpturing being, the curious fact, that at the head of Sydney harbour rude sculpturings of the kangaroo, &c., had been found cut on the rocks, when the turf was removed in building operations there. Mr Graham had likewise informed him that at the Cape, the Bushmen, one of the rudest existing races of humanity, live much in caves, and constantly paint on the walls of them the animals in their neighbourhood, and sometimes battle and hunting scenes, always in profile. Mons. Lartet has lately shown that the caves of Perigord have been inhabited by archaic man, at a time when apparently he had no metallic weapons, when the reindeer still inhabited the south of France, and when even the dog was not yet a domestic animal. Yet amongst the relics found in these Perigord caves have been discovered sculpturings upon stone, bone, and ivory, of different animals; and latterly a rude sketch of the mammoth itself. All this entitles us to hope that, if these cave researches are prosecuted, we may yet find on the cave walls sculpturings done by man in the most ancient times, and containing fragments of his earliest history.

[Dr Simpson's communication was illustrated by numerous drawings of the Fife cave sculptures, made by Mr Drummond, R.S.A., and Dr Paterson of Leith.]

3. Observations on New Lichens and Fungi from Otago, New Zealand. By Dr Lauder Lindsay.

The paper consists mainly of the author's observations on the microscopic anatomy of the reproductive organs (with illustrative figures) of the following *new species and varieties of Lichens and Fungi* collected by him in the province of *Otago*, New Zealand, in 1861:—

I. Lichens.

1. Abrothallus Curreyi, *Linds.*
2. Lecidea Otagensis, *Nyl.*
3. L. flavido atra, *Nyl.*
4. L. melanotropa, *Nyl.*
5. L. amphitropa, *Nyl.*
6. L. leucothalamia, *Nyl.*
   and *var.* melachroa, *Nyl.*
7. L. allotropa, *Nyl.*
8. L. coarctata, *Ach.*
   var. exposita, *Nyl.*
9. L. trachona, *Nyl.*
   var. marginatula, *Nyl.*
10. Lecanora homologa, *Nyl.*
11. L. peloleuca, *Nyl.*
12. Lecanora thiomela, *Nyl.*
13. Placopsis perrimosa, *Nyl.*
14. Opegrapha subeffigurans, *Nyl.*
15. O. agelæoides, *Nyl.*
16. O. spodopolia, *Nyl.*
17. Arthonia platygraphella, *Nyl.*
18. Platygrapha longifera, *Nyl.*
19. Pertusaria perfida, *Nyl.*
20. P. perrimosa, *Nyl.*
21. Pannaria immixta, *Nyl.*
22. P. gymnocheila, *Nyl.*
23. Psoroma sphinctrina, *Mnt.*, and
    var. pholidotoides, *Nyl.*
24. Physcia plinthiza, *Nyl.*
25. Ricasolia herbacea, *DN.*
    var. adscripta, *Nyl.*
26. Sticta subcoriacea, *Nyl.*
27. S. episticta, *Nyl.*
28. S. filix, *Hffm.*, and
    var. parvula, *Nyl.*
29. S. damæcornis, *Ach.*
    var subcaperata, *Nyl.*

These additions to the Lichen Flora of New Zealand and of the world may be thus tabulated :—

	Tribe.	Genus.	No. of New Species and Vars.	
I.	Lecideaceæ,	Lecidea,	8	9
		Abrothallus,	1	
II.	Lecanoreaceæ,	Lecanora,	3	9
		Pertusaria,	2	
		Pannaria,	2	
		Placopsis,	1	
		Psoroma,	1	
III.	Parmeliaceæ,	Sticta,	4	6
		Ricasolia,	1	
		Physcia,	1	
IV.	Graphideaceæ,	Opegrapha,	3	5
		Platygrapha,	1	
		Arthonia,	1	
		Total,		29

In the "Flora Novæ-Zelandiæ" of Dr Hooker there is no record of any species of *Opegrapha*, *Platygrapha*, *Umbilicaria*, or *Abrothallus*; and Professor Churchill Babington therein specially remarks, indeed, on the supposed absence of the genera *Opegrapha* and *Umbilicaria* (*Gyrophora*) from the Lichen-Flora of New Zealand. Subsequent researches have, however, shown that these genera are really represented, their apparent absence having been due probably to their not having been specially looked for. The foregoing list exhibits three species of *Opegrapha*, all new, being all the *Opegraphœ* contained in the author's collection; and Dr Knight and Mr Mitten have described other three species,* two of them new, as occurring in the North Island. The author's Otago collection also contains at least one species of *Umbilicaria*—a British one—*U. polyphylla*, L., and he has little doubt others will be discovered when the Lichens of the New Zealand *Alps* are specially collected.

II. Fungi.

1. Sphæria Lindsayana, *Currey*.
2. S. Otagensis, *Linds*.
3. S. Martiniana, *Linds*.
4. S. perrugosaria, *Linds*.
5. S. Cargilliana. *Linds*.
6. S. Usnearia, *Linds*.
7. Sphæria vermicularia, *Linds*.
8. S. Ramalinaria, *Linds*.
9. S. Stictaria, *Linds*.
10. Nectria armeniaca, *Currey*.
11. Æcidium Otagense, *Linds*.

Of these, Nos. 4 to 9 are types of a group, which is as yet virtually unknown to Fungologists, viz., the *Fungi parasitic on Lichens*; while No. 11 possesses interest in connection with the deformities or diseases it produces in the Phænogamous plant which it affects.

4. Orthogonal Isothermal Surfaces. By Professor Tait.

The Council reported that they had awarded the Keith Prize for the biennial period 1863-65, to Principal Forbes, St Andrews, for his "Experimental Inquiry into the Laws of Conduction of Heat in Iron Bars," which was printed in the last Part of the Transactions of the Society.

The Council also reported that they had awarded the Neill Prize for the triennial period 1862–65 to **Andrew Crombie Ramsay**, F.R.S., Professor of Geology in the Government School of Mines, and Local Director of the Geological Survey of Great Britain, for his various works and memoirs published during the last five years, in which he has applied the large experience acquired by him in the Direction of the arduous work of the Geological Survey of Great Britain to the elucidation of important questions bearing on Geological Science.

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* "Contributions to the Lichenographia of New Zealand," Trans. Linnean Soc. vol. xxiii. p. 101.

The following Gentlemen were duly elected Fellows of the Society:—

Fraser Thomson, M.D., Perth.
John M'Culloch, Esq.
T. Grainger Stewart, M.D., F.R.C.P.E.
Joseph M. Joseph, M.D., F.R.C.P.E.
Colonel Sir James Alexander of Westerton.

The following Donations to the Library were announced:—

Transactions of the Royal Irish Academy. Antiquities, Vol. XXIV. Parts 2-4. Science, Vol. XXIV. Parts 4 and 6. Literature, Vol. XXIV. Part 2. Dublin, 1864–65. 4to.—*From the Academy.*

Proceedings of the Royal Irish Academy. Vols. VII., VIII., IX. Part 1. Dublin, 8vo.—*From the Academy.*

Transactions of the Bombay Geographical Society. Vol. XVII. Bombay, 1865. 8vo.—*From the Society.*

Catalogue of the Printed Books in the Advocates' Library. Part II. Edinburgh, 1864. 4to.—*From the Library.*

On the Mechanical Principles of the Action of Propellers. By Professor W. J. Macquorn Rankine, LL.D. 4to.—*From the Author.*

Observations on the Functions of the Liver. By Robert M'Donnell, M.D. Dublin, 1865. 8vo.—*From the Author.*

Monthly Notices of the Royal Astronomical Society. Vol. XXVI. No. 1. London, 1865. 8vo.—*From the Society.*

Proceedings of the Royal Geographical Society. Vol. X. No. 1. London, 1865. 8vo.—*From the Society.*

Monday, 15*th January* 1866.

Sir DAVID BREWSTER, President, in the Chair.

The following Communications were read :—

1. Notes for a Comparison of the Glaciation of the West of Scotland with that of Arctic Norway. By Archibald Geikie, Esq., F.R.S.

In the course of the detailed investigations which, during the past six or seven years, have been carried on by the officers of the Geological Survey into the history of the glacial period in Britain, the desire naturally arose to compare the phenomena of glaciation so familiar in this country with those of some other region where they might be linked on to the action of still existing glaciers. No other part of Europe offered so many facilities for such a comparison as were to be found in Scandinavia. It was accordingly planned by my colleague, Dr John Young, and myself, to visit Norway in the summer of 1863. Unforeseen circumstances delayed the journey, and ultimately deprived me of the companionship of my friend. Nor was it until June of last year (1865) that, accompanied by two of my associates in the Geological Survey, Mr W. Whitaker and Mr James Geikie, I reached the Arctic Circle. All the Norwegian observations recorded in this paper were made conjointly with these companions.

The objects proposed to be accomplished in this excursion were—to compare, as minutely as time would allow, the ice-marks on the rocks of Scotland with those on the rocks of Scandinavia; to ascertain, from personal exploration, how far the glaciation of the Norwegian coasts and fjords could be traced to the action of land ice or of floating bergs; to trace, if possible, the connection between the ancient ice-work and the work of the living glaciers; and, generally, to watch for any facts that might help to throw light upon the history of the glacial period in the British Isles. Having only a few weeks at our disposal, we were far from aiming at original discovery in Norwegian geology. The main features of the disposition of the snow-fields and glaciers had already been given in the masterly sketch of Principal Forbes—a work which was of inestimable value to us.* More detailed descriptions of parts of the glaciation of Norway had been published by Scandinavian geologists—Esmark,† Hörbye,‡ Kjerulf,§ Sexe,|| and others. Yet I was not without the hope that, besides adding to our own experience, we might also be fortunate enough to find in the Norwegian fjords materials for making still more clear the geological history of our own western sea-lochs.

The close resemblance between the general outline of Scotland and that of Scandinavia is too well known to need more than a passing allusion. The numerous deep and intricate indentations, the endless islands and skerries, the mountainous shores, the host

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* "Norway and its Glaciers." 8vo. 1853. Mr Chambers also has referred to the striated rocks in different parts of Norway in his "Tracings of the North of Europe." 1850.

† Esmark. "Edin. New Phil. Journal," vol. ii. p. 116 *et seq.* (1826). In this paper the former presence of land ice over large areas from which it is now absent, and its powerful influence as a geological agent of abrasion, are, for the first time, distinctly recognised. The illustrations are taken from the south of Norway.

‡ Hörbye. "Observations sur les phénomènes d'erosion en Norvége."—*Programme de l'Université de Christiania pour* 1857. The author gives a careful *resumé* of all the observations made by himself and others upon the direction of the striæ on the rocks of Norway, and adds a number of maps, one of which shows the outward radiation of the striæ from the central mountain mass of Scandinavia. Yet he commits himself to no theory as to the nature of the agent by which the striæ were produced. In a concluding section upon the glacial theory, he says:—"Il est vrai sans doute qu'en général la direction des stries est parallèle à l'avancement des glaciers actuels; mais je ne vois pas que cette circonstance puisse suffisamment démontrer que les stries ont été gravées par les glaciers." "Je me joins à cette conclusion, que les sulcatures du Nord se presentent comme des produits d'un agent plus puissant et plus général que les glaciers dont l'action conserve toujours un caractère plus local." But he does not indicate what this more powerful and more general agent may be.

§ Kjerulf. "Über das Friktions-Phaenomen." Christiania. 8vo. 1860. See also Programme de l'Université de Christiania pour 1860, and Zeitschrift der Deutsch. Geol. Geselschaft, 1863, p. 619, and plate xvii.

|| Sexe. "Om Sneebræen Folgefon." Christiania. Universitetsprogram for andet Halvaar 1864. This paper gives a detailed account, with map and sections, of the Folgefon snow-field and its glaciers, including the well-known glacier of Bondhuus.

of short independent streams on the western coast; and on the eastern side, the broad undulating lowlands sending their collected drainage into large rivers, which enter the sea along a comparatively little embayed coast-line, are familiar features on the maps of both countries. This general outward semblance, which at once arrests the attention of every traveller in Norway, to whom the scenery of the western Highlands is familiar, depends upon a close similarity in the geological structure of the rocks, and a coincidence in the geological history of the surface of the two regions. Norway, from south to north, is almost wholly made up of metamorphic rocks, not all of the same age, yet possessing a general similarity of character. In like manner, the west of Scotland, from the Mull of Cantyre to Cape Wrath, is in great measure built up of gneiss, schist, slate, quartz-rock, granite, and other metamorphic rocks, quite comparable with these of Norway.*

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* My friend Herr Tellef Dahll, who, in conjunction with Dr Kjerulf, is carrying on the Government Geological Survey of Norway, wrote down for me the following order of superposition of the rocks of the south of Norway. He was not at the time acquainted with the order of succession in the north-west of Scotland, and expressed his surprise and pleasure to find that it corroborated so well the order established by his colleague and himself. I place in parallel columns the Norwegian and Scottish rocks, to show the general parallelism, without wishing to insist that the equivalents suggested here are in each case strictly exact.

Norway.		*N. W. Highlands.*
Devonian ?		Lower Old Red Sandstone.
Upper Silurian.		"
Lower Silurian. Hornblende, schist, gneiss, &c. Quartzite.	Lower Silurian	Schists, gneiss, and gneissose and schistoze rocks. Quartz-rock and limestones.
Schists with *Dictyonema norvegica*.		
Sparagmite and schists, about 2000 ft.		? Slates of N.-W. of Islay = Lingula flags ?
Red Sandstone and conglomerate occasionally present here. These and the overlying strata rest quite unconformably upon the		Red Sandstone and conglomerate (Cambrian) lying unconformably on the
Tellemarken formation—a vast succession of metamorphic rocks.		Fundamental or Laurentian gneiss.

This parallelism may require considerable modification, but it is at least

Besides the external resemblance due to the lithological nature of the rocks, beneath there is a still further likeness dependent upon similarity, partly of geological structure, and partly of denudation. Many of the Scottish sea-lochs have had their trend determined by lines of strike or of anticlinal axis, and the same result seems to have taken place in Norway. In other cases, the lochs and glens of the one country, and the fjords and valleys of the other, cannot be traced to any determining geological structure, but must be referred to the great process of denudation which has brought the surface to its present form.* In short, Norway and the Scottish Highlands seem to be but parts of one long table-land of palæozoic (chiefly metamorphic) rocks. This table-land must be of venerable antiquity; for it seems to have been in existence, at least in part, as far back as the Lower Old Red Sandstone. Since that time it has been sorely defaced by long cycles of geological revolution; rains, rivers, ice, and general atmospheric waste, have carved out of it the present valleys, and to all this surface change must be added the results of dislocations, as well as unequal upheavals and depressions of the crust of the earth beneath. Nevertheless, it still survives in extensive fragments in Norway, where it serves as a platform for the great snow-fields, while it can even yet be traced along the undulating summits of the mountains of the Scottish Highlands. One of its latest great revolutions was a submergence towards the west, which extended from the coasts of Ireland to the north of Norway, and gave rise to some of the most distinctive features of that part of Europe. No one can attentively consider the maps of the countries between the headlands of Connaught and the North Cape, without being convinced that the endless ramifying sea-lochs and fjords, kyles and sounds, were once land valleys. Each loch and fjord is the submerged part of a valley, of which we still see the upper portion above water, and the

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important at present in showing that, both in Norway and in Scotland, there is a bottom gneiss covered unconformably by strata containing fossils, and that these strata are again overlaid by an upper and later series of metamorphic rocks of Lower Silurian age.

* I have tried to trace the history of this process in the case of the Scottish Highlands, and I may be permitted to refer to "The Scenery of Scotland, viewed in connection with its Physical Geology," chap. vi.

sunken rocks and skerries, islets and islands, are all so many relics of the uneven surface of the old land. The indented form of the coast-line of the west of Scotland and of Norway is not evidence of the unequal encroachments of the sea, but is due to a general submergence of the west side of the two countries, whereby the tides have been sent far inland, filling from side to side ancient valleys and lakes.* Subsequent re-elevations are marked along both the Norwegian and Scottish shores by successive terraces or raised beaches.

But to one who has sailed and boated among the sea-lochs of Scotland, no feature of the Norwegian coast is at once so striking and so familiar as the universal smoothing and rounding of the rocks, which is now recognised as the result of the abrading power of ice. Every skerry and islet among the countless thousands of that coast-line is either one smooth boss of rock, like the back of a whale or dolphin, or a succession of such bosses rising and sinking in gentle undulations into each other. Such, too, is the nature of the rocky shore of every fjord; the smoothed surface growing gradually rougher, indeed, as we trace it upward from the sea-level, yet continuing to show itself, until at a height of many hundred feet it merges into the broken, scarped outlines of the higher mountain sides and summits.† In short, as is now well known, the whole of the surface of the country, for many hundred feet above the sea, has been ground down and smoothed by ice.

We sailed along the coast of Norway, between Bergen and Hammerfest, by the usual steamboat route, touching at many stations by the way, threading the narrow kyles and sounds that lie among the innumerable islands, and now and then running inland up some fjord far into the heart of the country. We halted here and there to spend a few days at a time in exploring some of the fjords and glaciers. What can be seen from the steamer on the coasting voyage is now familiar from the numerous descriptions which have been given of it in recent years. I shall therefore content myself with offering to the Society an account of two excursions to some distance from the ordinary route.

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* See a fuller statement of this subject in "Scenery of Scotland," pp. 125–137.

† The singularly ice-worn aspect of the Norwegian coast, as well as its strong resemblance to the west coast of Scotland, was succinctly described by Principal Forbes, "Norway and its Glaciers," p. 42 *et seq.*

A little to the north of the Arctic Circle lies the island of Melö, one of many which are here crowded together along the coast. It is only noticeable, inasmuch as it is a station at which the steamers call, and from which the great snow-fields of the Svartisen or Fondalen may be most easily visited. Here, as along all the rest of the Norwegian coasts, we find ourselves among bare bossy hummocks of rock thoroughly ice-worn. From the higher eminences the eye sweeps over the countless islets and skerries, and far across the Vest Fjord to the serrated peaks of the Lofodden Islands, which from the distance seem deep sunk in the north-western sea. The whole of the lower grounds is one labyrinth of *roches moutonnées*, raising their smooth backs like so many porpoises out of the sea, and out of a flat expanse of green pasture and dark bog which here covers an old sea-bottom. The striations and groovings are still fresh on many of the smoothed surfaces of gneiss, and invariably run straight out to sea in the line of the long valley up which the sea winds inland among the snowy mountains. It cannot be doubted that a vast mass of ice has come seawards down this valley, and that all these ice-worn hummocks of rock were ground down by it. The wide valley or opening which stretches inland from Melö, is formed by the converging mouths of a number of narrow fjords. Of these the most northerly is the Glommens Fjord, which is bounded along its northern side by a range of high mountains, with a serrated crest and abundant snowy clefts and corries. Southward lies a belt of lower ice-worn hills, cut lengthwise by the Bjerangs Fjord, and bounded on the south by the Holands Fjord, on the south side of which rises another range of scarped snow-covered mountains.*

From the *gaard* of Melö we boated eastwards among various small islets and channels, passing soon into the Holands Fjord, up which we continued until we rested underneath the great snow-field and glaciers of Svartisen. In this excursion we started from the coast, amid islands, all moulded, like those of the west of Scotland, by the ice of the glacial period, and we ended among rocks on which the present glaciers are inscribing precisely the same markings. One of the first features which arrested attention was the contrast between the smoothed, ice-worn surface of the lower grounds and the craggy, scarped outlines of the mountain crests.

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* Although I use the word *mountains*, there is no definite system of ridges; on the contrary, these fjords must be regarded as indentations along the edge of a great table-land, of which the average level may range from 3000 to 4000 feet above the sea, and which serves as the platform on which the wide snow-fields lie. See "Norway and its Glaciers," pp. 190, 232.

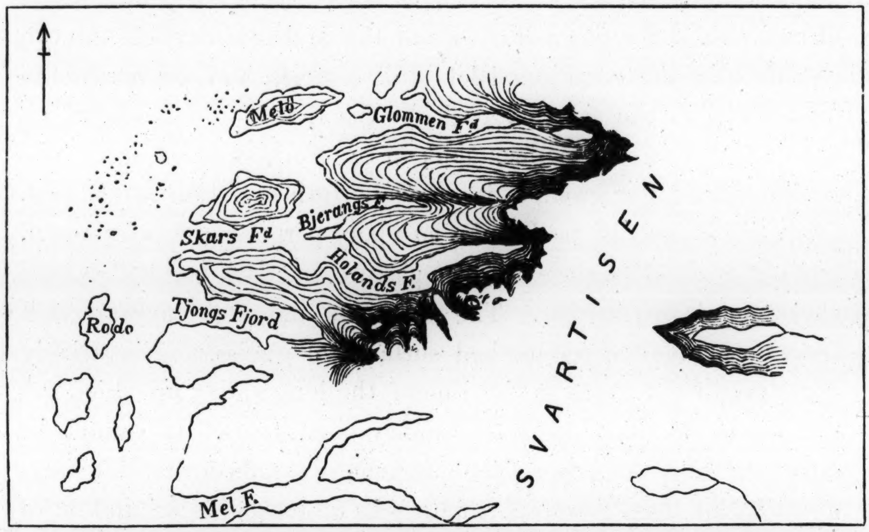


Fig. 1.—Map of the Neighbourhood of the Holands Fjord (Munch).

This was especially marked along the northern side of the Glommens Fjord, where the ice-worn rocks form a distinct zone along the side of the rough, craggy hills. To the north of Melövaer this ice-worn belt was estimated by aneroid to rise about 200 feet above the sea. Its smoothed rocks are abundantly rent along lines of joint and other divisional planes; their ice-worn aspect must thus be imperceptibly fading away. The rough rocks above them sometimes show traces of smoothed surfaces, as if they too had suffered from an older glaciation, of which the records are now all but obliterated. The line of division between the belt of rocks which have been smoothed by ice, and those which have been roughened and scarped by atmospheric waste, slopes gently upward in the direction of the central snow-fields of the interior. While at Melövaer it seemed to rise only about 200 feet above the sea; at Fondalen, twenty-five or thirty miles inland, it mounts to a height of fully 1500 feet. A tract of bare hills, lying between the Glommens and the Holands Fjord, and rising eastward into the snow-covered table-land, is well smoothed in the direction of these fjords. In short, the whole of the broad depression between the two fjords has been filled with ice, moving steadily downwards from the snow-fields to the sea.

It was interesting to watch, on every little islet and promontory under which we passed, the same details of glaciation so familiar along the margin of our Scottish fjords. The rocks are, as usual, smoothed into flowing lines, and slip sharply and cleanly into the water. They are well grooved and striated, these markings differing in no respect from those in Britain. Moreover, it was easy to see that the ice which had graven these lines must have moved down the fjord, for the *lee* or rougher side of the crag looked seawards. It was likewise clear that the scorings were not the work of drifting bergs or coast ice, for they could often be seen mounting over projecting parts of the banks, yet retaining all the while their sharpness, parallelism, and persistent trend. Another point of similarity to west Highland scenery, was found in the strange scarcity or absence of drift and boulders. I do not mean to assert that these are not to be met with at all, but they do not exist so prominently as to catch the eye even of one who is on the outlook for them. The rock everywhere raises its bare knolls to the sun as it does on the coasts of Inverness and Argyll. To complete the resemblance, the Norwegian fjord has its sides marked by the line of a former sea-margin, about 250 feet above the present. This terrace winds out and in among all the ramifications and curves of the fjord, remaining fresher and more distinct than the raised beaches of the west Highlands usually are, and even rivalling one of the parallel roads of Lochaber.

We rested for a week at the hamlet of Fondalen, on the south side of the Holands Fjord. It stands at the mouth of a deep narrow valley on the line of the terrace, which here runs along the crest of a steep bank of rubbish covered with enormous blocks of rock—an old moraine thrown across the end of the valley. There seems to have been at one time a lake behind this bank, formed by the ponding back of the drainage of the valley, and gradually emptied as the outflow-stream deepened its channel through the moraine.

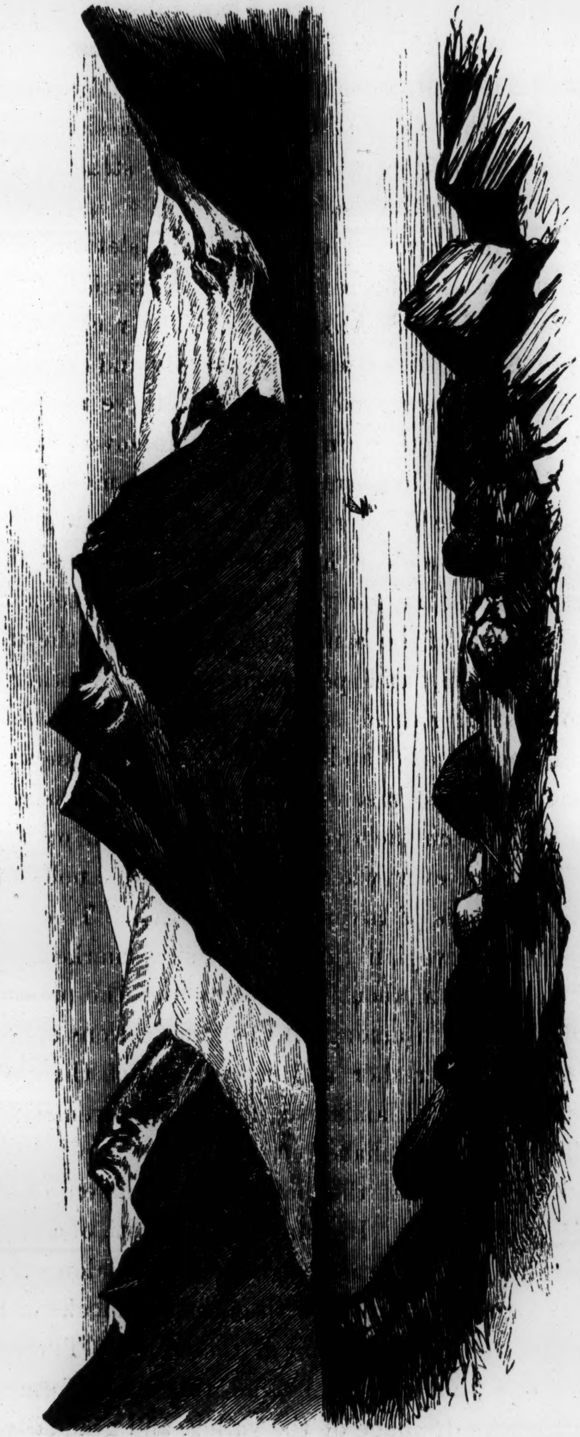


Fig. 2.—View of the two Glaciers of Fondalen, Holands Fjord.

At the head of the valley a small glacier descends from the snow-field of Svartisen. There could be no better locality for studying the gradual diminution of the glaciers, and for learning that it was land-ice that filled the Norwegian fjords, over-rode the lower hills and mountains, and went out boldly into the Atlantic and Arctic Sea. The Holands Fjord runs, as I have said, approximately east and west, and this short narrow valley descends from the south. The fjord was filled with ice, and is therefore polished and striated along the line of its main trend. The valley of Fondalen was likewise filled with ice, moving down to join the mass in the fjord; and its rocks, too, are striated in the length of the valley, or from south to north. The moraine of Fondalen is a proof that a glacier once descended to the Holands Fjord at that point. Farther evidence is found in the fact, that the sides of the valley are ground and striated for 700 feet and more above its bottom. Moreover, these polished and scored rocks can be traced up to and underneath the glacier. I crept for some yards under the ice, and found the floor of gneiss on which it rested smoothly polished and covered with scorings of all sizes, exactly the same in every respect as those high on the sides of the valley, in the fjord below, and away on the outer islands and skerries. Over this polished surface trickled the water of the melted ice, washing out sand and small stones from under the glacier.

We climbed the steep eastern side of the valley above the foot of the glacier, and found the hummocks of gneiss wonderfully glaciated up to a height of fully 700 feet. The gnarled crystalline rock has been ground away smoothly and sharply, so as to show its twisted foliation, as well as the patterns of a marble, are displayed on a polished chimney-piece. Even vertical or overhanging faces of rock are equally smoothed and striated. Many of the *roches moutonnées* are loaded with perched blocks of all sizes, up to masses 30 or 40 feet long. Above the limit to which we traced the work of the ice, the rocks begin to wear a more rugged surface, until along the summit of the ridges they rise into serrated crests and pinnacles. This rougher outline is, of course, the result of atmospheric waste, guided by the geological structure and chemical composition of the rocks.

The glacier descends from the snow-field, which was guessed to have there an elevation of about 3500 feet, to a point in the valley about 400 feet above the sea. The distance from the snow-field to the foot of the glacier looks not much more than one English mile—at least it is but short, compared with the rapidity of descent. Hence the glacier is steep, and in some places much crevassed. Issuing from the upper snow, in a steep, broken, and jagged slope of blue ice, it descends by a series of steps, till, getting compacted again in the valley below, it passes into a solid, firm glacier, with a tolerably smooth surface, forming a declivity of 12° or 15°.

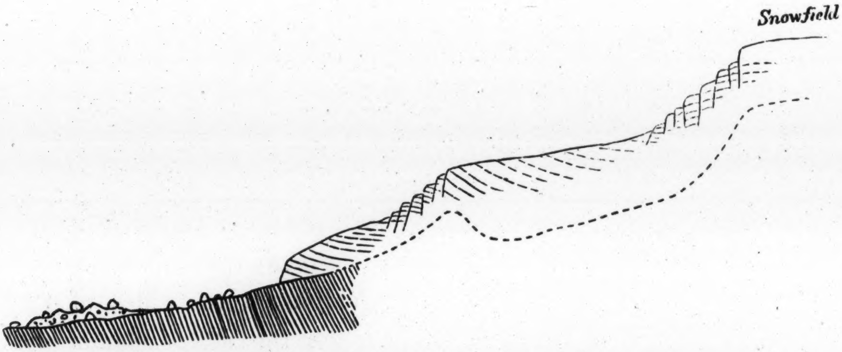


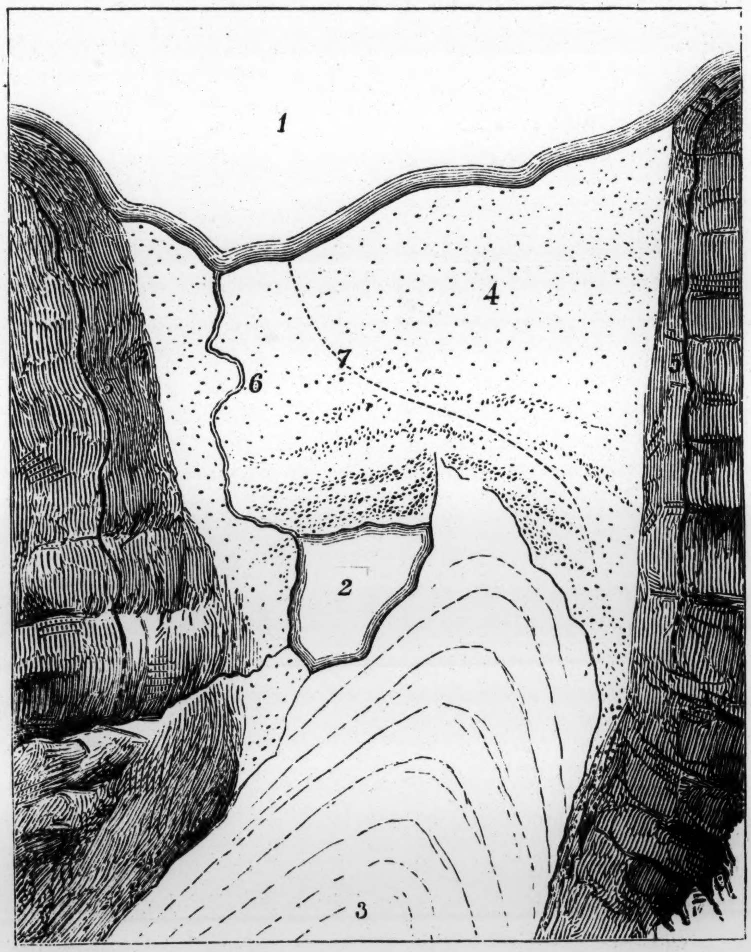
Fig. 3.—Longitudinal Section of smaller Glacier. Fondalen.

At a point about half a mile or less from the foot of the glacier the valley suddenly contracts, and the glacier, much narrowed and compressed, tumbles over a second steep declivity in a mass of broken ice. The crevasses speedily unite, and after another descent of 300 or 400 yards at an angle of 25°, the glacier comes to an end. At the point where the strangulation takes place, the glacier lies in a kind of basin, of which the lower lip presents proofs of the most intense erosion. On the western bank, in particular, a mass of the mountain side which projects into the ice has been ground away, and shows plainly enough, by its form and striæ, that the glacier, ascending from the basin, has climbed up and over this barrier, so as to tumble down its northern or seaward side.

The course of this little glacier is now too short to admit of the formation of moraines. Yet there are large heaps of rubbish and enormous masses of rock scattered over the valley below, and the moraine at Fondalen is a further proof that when the ice formerly filled the valley, its surface received abundant detritus from the mountain slopes on either side.

Opposite Fondalen, the Holands Fjord, passing through a deep and narrow channel on its northern bank, trends in an east-north-easterly direction, but just before taking this course, it sends eastward a bay which terminates at the mouth of a valley about a mile above the hamlet. This valley is considerably larger than that just de-

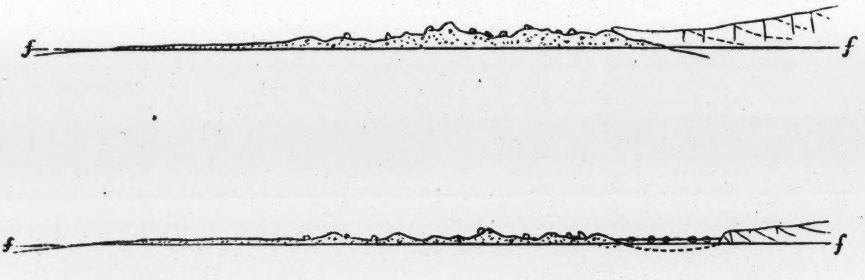
Fig. 4.—Sketch-map of lower end of larger Glacier. Fondalen.



1. Holands Fjord. 2. Small lake at end of glacier. 3. Glacier. 4. Alluvial plain rising into moraine mounds as it approaches the foot of the glacier. 5. Line of marine terrace or "raised beach," about 250 feet above sea-level. 6. Present course of stream. 7. Old course of stream.

scribed, and it is occupied by a much longer and larger glacier. To one who looks up the valley from the opposite side of the fjord, it seems as if the ample glacier which fills up the bottom sweeps down from the snow-field in a rapid descent to the very edge of the sea. On a visit to the locality, however, it is found that between the foot of the glacier and the sea margin, there lies a plain of shingle and alluvium, which is partly covered with a brushwood of birch, and partly with a scanty pasturage. As it nears the ice it rises into ridges and hummocks, which increase in size as we ascend. These are true moraine mounds, rising often 60 or 70 feet above their base, and strongly reminding me of the moraines at Loch Skene, Peeblesshire. They consist of earth and stones, and are strewed over with large blocks of gneiss, porphyry, limestone, and other crystalline rocks. About a quarter of a mile from the margin of the fjord, along the eastern half of the breadth of the valley, these mounds come in contact with the foot of the glacier, which is there pushed in a long tongue down the valley. The ice overrides the moraine heaps, ploughing them and pushing them over.

Fig. 5.—Sections across the lower end of the larger Glacier. Fondalen.



In the upper section, the glacier is shown overriding its moraine; in the lower, the small lake with floating ice intervenes between the end of the glacier and the moraine. In each section *f f* marks the level of the fjord.

On the west side of this prolongation of the glacier, the ice is separated from the moraine mound by a small lake, of which the surplus waters find their way seaward by cutting through the moraine. Like Loch Skene and many lakes still existing in Britain, this sheet of water is formed by the dam of rubbish thrown down by the glacier across the valley. It is full of fragments of ice, which break off from the parent mass, and float across to the north or lower side, where they strand on the moraine heaps, and gradually melt away. The smaller pieces, however, often find their way into the stream by which the lake discharges itself, and are then carried down into the fjord. From the mean of several observations taken with the aneroid, I estimated the surface of this lake to be about 25 feet above the level of high water in the fjord. We had no means of measuring its depth, yet, from the slope of the glacier, it may be inferred that the bottom of the ice is probably lower than the level of the sea.

Proofs that the glacier was once much larger than it is now, may be well seen on the west side of the valley, a little above the lake. The shelving slopes of the mountain for several hundred feet upward have been shorn smooth, grooved, and striated, and every polished hummock of rock is loaded with huge fragments of stone, and heaps of earth and angular rubbish. Here, as at every glacier we visited, the glaciation of the rocks was exactly similar, down to the minutest detail, with that of the coast and outer islets, as well as with that of the Scottish glens and sea-lochs.

But the feature which most interested us was the relation of this large glacier of Fondalen to the marine deposits of the locality. The foregoing sketch (fig 4.) shows that the high terrace so marked along the sides of the Holands Fjord enters this valley, and extends on the western mountain side, at least, as far as the foot of the glacier. Hence the gravelly plain and the moraine mounds that separate the glacier from the fjord are overlooked on either side by a raised sea-beach. In examining attentively the nature of the material of which the mounds nearest the glacier were composed, we were struck with its difference from the loose, coarse character of the ordinary moraine rubbish, and its resemblance to the upper boulder-clay of Scotland. The glacier is pushing great noses of ice into and over those mounds, so that freshly exposed sections are abundant. The deposit is a loose sandy clay or earth full of stones, among which the percentage of striated specimens is not large. The larger blocks of gneiss and schist appeared to us not to occur in this clay, but to be tumbled down upon it from the surface of the glacier. We had hardly begun to look over a surface of the clay, ere we found fragments of shells, and in the course of a few minutes we picked up several handfuls, chiefly of broken pieces of *Cyprina Islandica*, but including also single valves of *Astarte compressa*, *&c.*

We even took out two or three fragments which were sticking in the ice of the glacier. These shells are not peculiar to one spot, but occurred more or less abundantly across the valley.

From the nature of the material of which these mounds consist, and from the occurrence of marine shells, it became evident that we were looking not merely upon ordinary moraine heaps—the detritus carried down on the surface of the ice and discharged upon the bottom of the valley. The glacier was engaged in ploughing up the sediment which had been formerly deposited in the valley by the sea, and on the heaps of earth and clay so formed were thrown the gravel and blocks brought down by the glacier. In short, we saw here actually at work a process of excavation, by which it had been conjectured that the marine drift was removed from certain valleys in the British Isles.*

We made two attempts, both unsuccessful, to climb to the vast table-land of snow from which these glaciers are fed. But we succeeded in reaching a point from which a good view of the seemingly boundless undulating plain of smooth snow could be obtained. We ascended the ridge that separates the two glacier valleys just described. After leaving the raised beach of Fondalen, with its massive erratics, we climbed a steep slope, clothed with a thick brushwood of birch, mountain-ash, and dwarf-willow, and luxuriant masses of ferns, bilberries, cloudberries, juniper, rock-geranium, lychnis, &c. The beech trees are often a foot or a foot and a-half in diameter at the base, and are the building material used at the hamlet of Fondalen below. These trees, at the height of 1320 feet above the sea, still often measure a foot across at the bottom, and 15 or 20 feet in height. At this height, and even considerably lower, there were large sheets of snow on the 12th of July, and these increased in number and depth as we ascended. The birch trees grow smaller and more stunted the farther they struggle up the bare mountain ridge, until they become mere bushes. The willows, in like manner, dwindle down till they look like straggling tufts of heather, though still bearing their full-formed catkins. At a height of 1690 feet, these stunted bushes at last give place to a scrub of bilberry, mosses, and lycopodia. The mountain consists of gneiss, sometimes massive and jointed, sometimes fissile and flaggy, with a strike towards W. 15° S. The extent to which the higher portions have suffered from the disintegrating effects of the weather is remarkable. The gneiss is split up along its joints into large blocks, which lie piled upon each other in heaps of angular ruin. We noticed one or two masses which differed in lithological character from the rocks around; these may possibly have been ice-borne from some of the neighbouring eminences. On reaching a point 2700 feet above the fjord, our further passage was arrested by a narrow, shattered, knife-edge of gneiss, along which it was impossible to advance. But from this elevated point we could judge of the general aspect of the great snowy table-land of the Svartisen, which was sloping towards us, while the two glaciers were spread out as in a plan beneath.

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* See Prof. A. C. Ramsay, "Glaciers of Switzerland and Wales," 2d edition p. 60.

The branch of the Holands Fjord, which opposite to the hamlet of Fondalen, strikes off to the north-east for seven or eight miles, is bordered on the south side, and closed in at its further end, by a range of steep, almost precipitous, walls of rock, the summits of which are on a level with, and indeed form part of the great table-land. Here, as in so many other parts of Norway, we are reminded that the fjords are, after all, mere long sinuous trenches, dug deeply out of the edge of a series of elevated plateaux. And, looking up to the crest of these dark precipices, we see the end of this snow-plain peering over, and sending a stream of blue glacier ice down every available hollow. We counted seven of these tiny glaciers, exuding like tears from under the snow, and creeping downward under the sombre cliffs of gneiss. None of them comes much below the snow line, and none, of course, reaches the sea. The largest of them is near the end of the fjord, and appears as a broken, crevassed mass of ice, moulded as it were over the steep hill side, and, when seen from below, seeming about to slip off and plunge into the fjord. Fragments of it are continually breaking away, and rolling, with the noise of thunder, and clouds of icy dust, down the shelving sides of the mountains. These glaciers are, for the most part, the icy drainage of the snow-field. But there are one or two lying in corries, and quite detached from the main snow-field, though probably connected with it in winter.

We left this delightful fjord not without regret, and catching again the coasting steamer at Melovaer, proceeded northwards. Between Melovaer and Bodö, the higher mountains have wonderfully craggy and spiry outlines, only their lower parts showing the smoothed contour of glaciation. But where the coast hills sank, as towards a fjord or bay, the ice-moulded forms could be traced to a greater height. To the north of Bodö, the contrast between the sharp weather-worn peaks above, and the flowing ice-worn hummocks and hill sides below, is singularly startling. Principal Forbes, who gave a characteristically faithful drawing to illustrate this feature, places the upper limit of glaciation at from 1500 to 2000.* We should have estimated it to be considerably lower. Through narrow kyles and intricate sounds, reminding one at every turn of detached portions of West Highland or Hebridean scenery, the steamer slowly wound its way, and then across the Vest Fjord to the Lofodden Islands. The weather now unfortunately proved unfavourable for geological observation. In sailing through the Rafte Sund, we saw what looked like moraines at the mouths of some of the valleys, and the lines of moraine terraces continued as marked as ever. Well ice-worn rocks were also observed at the openings of some of the valleys, but we were rather impressed with the general ruggedness and absence of glaciation among the Lofoddens.

To the north of Tromsö lies the island of Ringvatsö, noticed by Mr R. Chambers.† The moraine which he describes as damming up the circular sheet of water, whence the island takes its name, really coincides with the line of the higher of the two strongly marked terraces or sea-margins of this part of the Norwegian coast. It thus illustrates the history of the moraine and terrace, below the smaller glacier at Fondalen. It was further interesting to mark that the glacier of Ringvatsö, partially hidden under snow, lies in a hollow or corry surrounded with precipices, and quite cut off from any snow-field. The accumulation of snow in the corry itself must thus be sufficient to give rise to the glacier. In looking at this island, I was again forcibly reminded of the history of the glaciers of Tweedsmuir and Loch Skene, where, on dimples of the hill tops, and in deep cliff-encircled recesses, there must have gathered snow enough to form streams of ice, which caught and carried on their surface the piles of rubbish and huge blocks of greywacke that now form the moraines of Midlaw, and dam back the waters of Loch Skene. A large snow-field is not necessary for the production of a glacier that may form comparatively extensive moraines.*

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* Norway and its Glaciers, p. 58.

† Tracings of the North of Europe, p. 145.

The south-western side of the Lyngen Fjord is formed by a mass of high ground, which shoots up steeply from the sea to a height of 4000 feet or more. Every hollow and cliff is smothered with snow, which descends in straggling streaks and patches almost to the edge of the water. We sailed up the fjord for some miles, and had a full view of this truly magnificent coast line. We counted from ten to twelve small glaciers nestling in separate corries, and also two or three on the north-eastern side. There was here the same evidence of the formation of glaciers in small independent hollows of the mountains, quite detached, at least in the summer, from any large snow-field.

We halted at the island of Skjaervö (lat. 70°) for the purpose of making an excursion across the Kvenangen Fjord and up the Jökuls Fjord, to see the glacier which was said to reach the level of the sea.† The metamorphic rocks among which the Jökuls Fjord lies, are for the most part of a flaggy quartzose character. Sometimes, especially where they are most fissile, they are violently crumpled. Parts of them pass into hornblende rock and actinolite schist. Their average strike is on an east and west line. They are much jointed, and yield freely to the action of the weather. Hence, a rough and angular surface has very generally replaced the ice-moulded outlines, though these still here and there remain.

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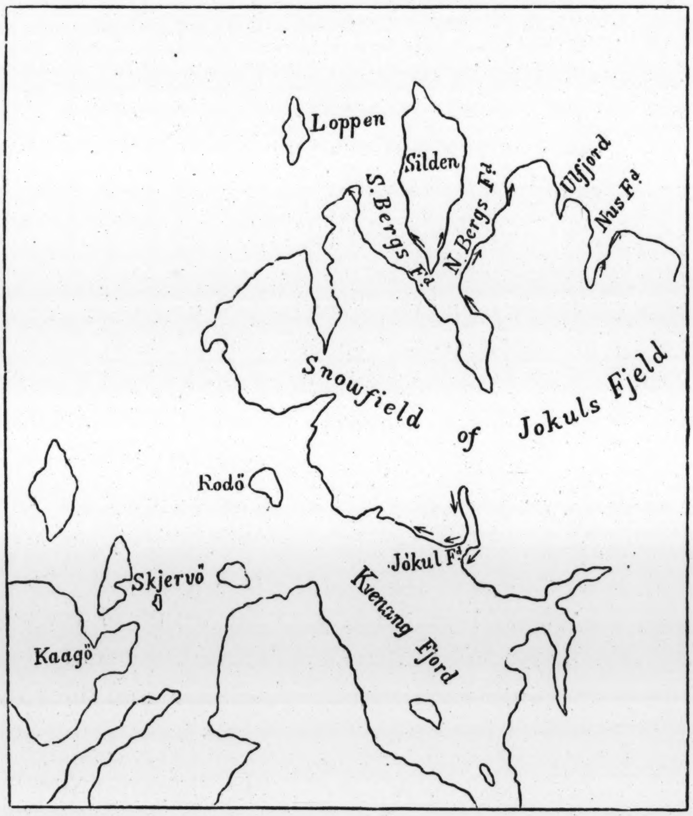
* North Wales presents a number of illustrations of this remark, such as Cwm Graianog, Cwm Idwal, &c. (see Professor Ramsay's Glaciers of North Wales).

† This glacier was noticed by Von Buch, and is mentioned by Principal Forbes. When we visited it, I was not aware that a brief account of it had been given in vol. ii. of "Peaks, Passes, and Glaciers," *second series*. Mr J. F. Hardy, the writer of that description, started overland from Talvik on the Alten Fjord, and reached the Jökuls Fjord below the glacier, to which he ascended by boat. Like my own party, he did not climb the glacier, but he seems to have regarded it as connected with the snow-field above. Though I did not succeed in ascending the rugged cliffs, I had no doubt that the lower glacier, from its colour, and the steepness and contraction of the gorge above it, is a true *glacier remanié*, and like the Suphelle glacier described by Forbes ("Norway and its Glaciers," p. 149), is quite disconnected, at least in summer, from the snow-fields above.

The same two prominent terraces already mentioned are well marked along the sides of the Jökuls Fjord. The lower one is about 60 feet, the higher about 152 feet (aneroid measurement) above high-water mark, and several others less distinct, occur between the higher and the sea. The upper is especially marked, often running as a shelf cut out of the rock. This feature was noticed along many parts of the Norwegian coast, even (as in the Jökuls Fjord) in sheltered places where wave action cannot be supposed ever to have been very strong. As the date of these rock-terraces probably goes back into the glacial period, it occurred to me that they may have been due in large measure to the effects of the freezings and thawings along the old "ice-foot," and to the rasping and grating of coast ice. Such, too, may have been the origin of the higher horizontal rock-terraces of Scotland.

Fig. 6.—Map of the Jökuls Fjeld promontory (after Munch). The arrows show the direction of the old ice striae.



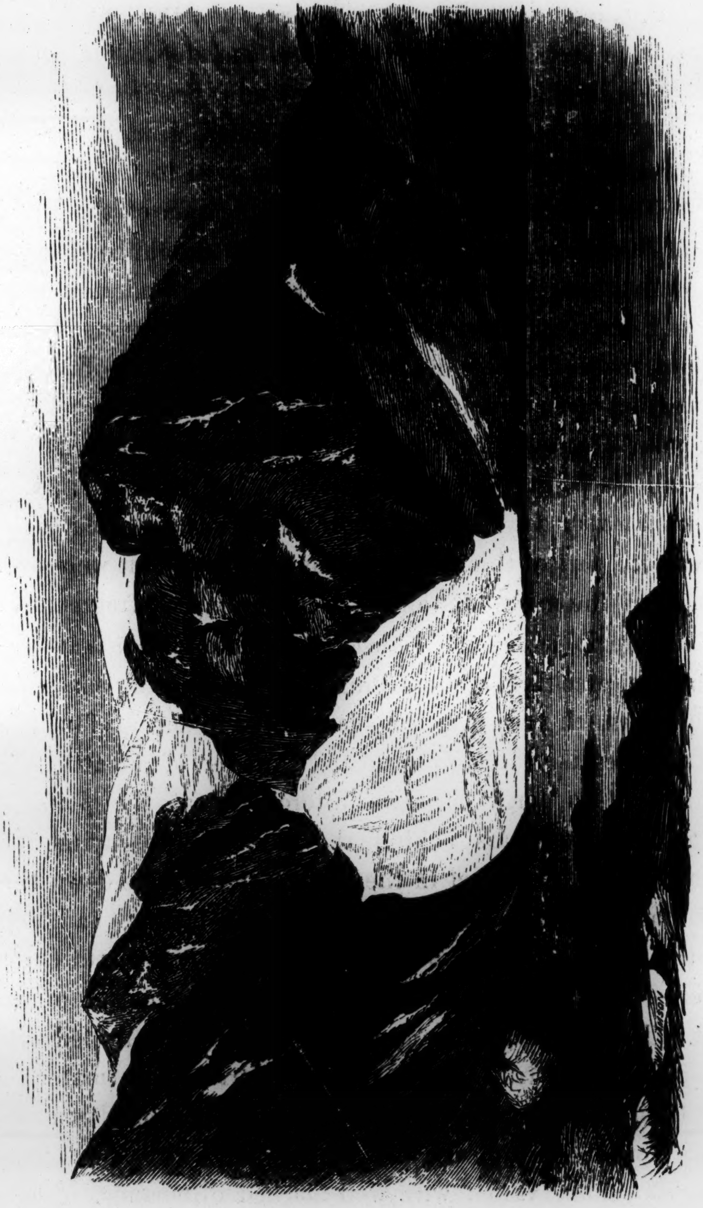
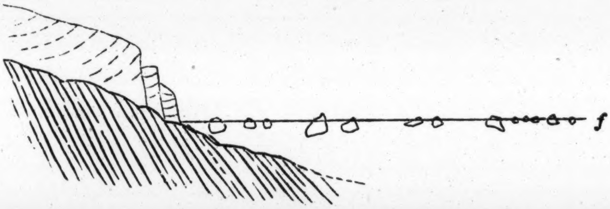


Fig. 7.—View of Jökuls Fjord Glacier.

At the head of the fjord the terraces disappear along the steep, bare sides of the mountains. A moraine mound of loose rubbish and large blocks lies on the west side, and extends a little way into the fjord, pointing towards a similar ridge on the opposite side, as if both were parts of a curved terminal moraine. The view from this ridge is singularly imposing. The sombre precipitous mountains sweep upward from the edge of the water, seamed everywhere with streaks and sheets of snow. Down even to the beach these snow-drifts lie; and it gives a vivid impression of the high latitude of the place, that even in July there should be deep masses of snow overhanging tangle covered rocks, and undermined by the wash of the waves. Over the crest of the mountains, at the head of the fjord, we see the edge of the great snow-field of the Jökuls Fjeld, and stealing down from underneath, the snow comes a broken, shattered mass of glacier ice, broadest at the top, and narrowing downwards till its point disappears in a deep cliff or ravine, perhaps a third of the way down from the surface of the snow-field to the sea. The eastern part of this glacier seems plastered as it were over the forehead of the mountain, and is ever sending off fragments down the dark precipice below. Indeed, the whole glacier is in constant commotion, cracking, and crashing, and discharging masses of ice and snow, which pour over the black rocks in sheets of white dust, with a noise like the unintermitted thunder of a battle. These ice-falls are in large measure intercepted at the point where the glacier disappears behind the side of the ravine. They seemed, indeed, to collect in the ravine, and to slide down through it; for at its lower end a second glacier begins, and expands with the expansion of the hollow in which it lies, till it reaches the edge of the fjord, where it may be a quarter of a mile broad. This lower glacier appeared to me not connected with the snow-field, but a true *glacier remanié*, deriving its materials entirely from the avalanches of snow and ice that pour down upon its surface from the precipices overhead. It has a white, or dull greenish white colour, varied with well-marked dirt-bands. The slope of its surface was judged to be fully 20° or 25°. A few longitudinal crevasses make their appearance along the middle of the glacier, and a little further down, the transverse crevasses increase in number and size, until at its foot the glacier, broken by large semicircular rents, becomes a tumbled mass of ruin. Last summer these cliffs of granular loose textured ice in some places overhung the waves. But the dark rock was likewise seen peering out along the water's edge, underneath the ice, which does not push its way out to sea in a mass, but ends abruptly where it meets the water. From these icy walls fragments and large slices break off, and fall either on the margin of rock or into the fjord, which is thus covered with hundreds of miniature icebergs, slowly drifted downwards against wind and tide, by the surface current of freshwater. This process is called "calving" by the natives, and so great is the commotion sometimes produced, that according to the information collected by Von Buch, the Lapp huts along the margin of the fjord are sometimes inundated by the waves propagated outwards from the falling masses. The floating fragments of ice look like little models of Arctic bergs; their forms are often singularly fantastic; they may be seen shifting their position, and even capsizing, as their submerged parts melt away; some of them carry stones and earth on their surface; and many are aground along the margin of the fjord, and rise and fall with the tide, or the ripple of the waves. We passed two or three which were from 8 to 10 feet long, and rose from 3 to 4 feet out of the fjord. Our boat grated against several, which seemed only a foot or two in size, yet the shock of the collision shewed how much larger was the portion concealed under water.

Fig. 8.—Section of Foot of Jökul's Fjord Glacier.



To the east of the upper glacier the snow-field sends another icy stream down the face of the shelving precipices which descend into a higher valley. We could hear the roar of the avalanches even when the glacier itself became hidden behind the intervening spur of the mountain. From the rocky declivities of the Jökul's Fjord also, stones were heard and seen bounding from point to point in their descent towards the long heaps of debris at the bottom. In short, in this lonely uninhabited spot, the activity and ceaselessness of the wasting powers of nature come before the traveller with a memorable impressiveness. The wide snow-field that seems to lie so sluggish and still among the distant mists, is yet seen to be in slow but constant motion, pushing its ice-streams towards the valleys, and grinding down the hard rocks over which it moves. Frosts, rain, and springs have scarped the shoulders of every mountain, and poured long trains of rubbish down its sides. And if this can be now done under the present climate of Norway, how much more powerful must the abrasion have been when the ice, in place of being arrested on the brow of the mountain, filled up the fjord, and pushed its way into the Arctic Sea.

From the open mouth of the Kvenangs Fjord, in the passage between Skjaervö and the Jökul, the outline of the neighbouring land is well seen. The steep, serrated ridge of the Kvenangs Tinderne shows its tiny glaciers nestling in corries both on its northern and southern sides. The sides of the Kvenangs Fjord are ice-moulded and striated in the direction of the inlet, and its islands are only large *roches moutonnées*. In looking back at the mountainous tract of the Jökul's Fjeld, we see that it is another snowy table-land jutting out as a promontory into the Arctic Sea, deeply trenched with long, narrow fjords, and pushing glaciers down every glen and hollow that descends from the plateau of snow.

We visited the north-western and northern sides of this snow-field, boating up the Bergs Fjord to the hamlet of that name, and after ascending to its glaciers, continuing our excursion by boat into the Nus Fjord. (See fig. 5.) In ascending the South Bergs Fjord, we found the gneissic and schistose rocks polished and striated from east to west, which is the direction of the inlet, and in turning into the North Bergs Fjord, which runs nearly at a right angle to the other, the striæ were seen to turn out of the Lang Fjord and bend northwards through the northern limb of the Bergs Fjord. At the hamlet of Bergsfjord these ice-mouldings are especially well shown, and there, as well as along many parts of the fjord, occur lines of rock-terraces, often strewed with quantities of angular blocks. Two of the most marked of these horizontal bars have an elevation of about 50 and 150 feet respectively.

Behind the hamlet the ground slopes up to a point about 250 feet above the sea, beyond which lies the mouth of a valley that runs up into the heart of the mountains. We climbed the terraced slope leading to this recess, and found that the lower half of the valley is occupied by a lake about a mile long, and said to be 30 fathoms deep. It lies in a rock basin, and the rocks around its margin show that they have been powerfully abraded by ice. We were told that three weeks before our visit this lake was solidly frozen over; great sheets of snow, indeed, still descended to the water's edge, and were melting away under the glare of a fierce July sun. At the far end of the valley mounds of angular rubbish, cumbered with huge blocks of stone, stretched from side to side, while overhead two glaciers came out of the edge of the snow-field, and hung down the steep mountain side—the longer one almost reaching the bottom of the valley. Here, too, the ice was ever breaking up and crashing down the precipices in clouds of snowy dust. The debris of ice gathered into talus heaps below, like the *cones de dejection* at the foot of a winter torrent.

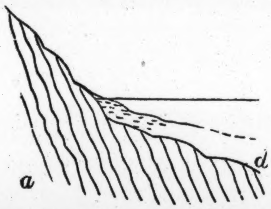
From Bergsfjord we continued our boating voyage down the fjord, and found fresh proofs that a vast body of ice, descending from the lofty Jökuls Fjeld, had moved northwards along the length of the inlet. Every promontory was beautifully smoothed and polished; while the grooves and striæ slanted up and over the projecting bosses of rock, as they do in Loch Fyne and the other western sea-lochs of Scotland. Round the headland at the mouth of the Bergs Fjord we turned eastward, and soon passed the mouth of the Ulfjord. We could see that, at the far end of that inlet, the snow of the great table-land moves outward to the edge of the dark precipices which encircle the Ulfjord, and actually forms on the crest of these precipices a white cliff, from which, of course, avalanches are constantly falling. Yet the under part of this snowy cliff is not snow, but ice, as shown by its blue colour contrasting with the whiteness of the upper layer, which is snow. At the foot of the precipice a glacier, derived probably in part, like that of Jökul's Fjord, from the ice-falls from above, creeps towards, but does not reach, the bottom of the valley. Continuing our eastward journey we saw the same terraces still skirting the hill sides, now as green platforms of detritus loaded with angular blocks, and now as sharp horizontal notches in the bare rocks. We were likewise struck here, as in other parts of the Norwegian coast, with the greater freshness of the ice-markings near the sea-level, when compared with those higher up—a difference which is likewise very noticeable in the west of Scotland.

The Nus Fjord is about six miles long, and lies between the Ulfjord and Oxfjord. Its margin is terraced by the same horizontal lines so constant in this region. Its south-western side presents a singularly arctic scene. A range of deeply cleft and embayed crags and precipices, plentifully streaked with snow, rises up to the edge of the snow-field, which, as usual, presses down every larger valley in a stream of blue ice. Eight or ten distinct glaciers may be counted, of which at least three descend from the snow-field. The others lie in corries detached from the snow-field, though in some cases connected with it by nearly perpendicular streaks of snow. Here, as in the Ulfjord, the edge of the great sheet of snow which covers the table-land may be seen ending off abruptly as a cliff upon the crest of a dark precipice of rock, and from the colour of the lower part of the cliff, it is plain that from pressure and motion, the under portion of the snow sheet is converted into ice, and as ice, reaches the verge of the table-land, where it breaks sharply off, and sends its ruins to the bottom of the precipice underneath. There the debris, mingled with the winter snow, is anew converted into solid ice, and creeps downward as a glacier.

At the head of the fjord, on the south-east side, the mouth of a valley which terminates inland at the foot of a glacier is blocked up by an old moraine. Behind this rampart of detritus the valley spreads out as an alluvial plain, evidently at one time a lake formed by the moraine barrier at the foot. The moraine itself is strewed with enormous angular blocks of rock, beside which the huts of a miserable Lapp encampment look like mere pebbles. The side of this moraine facing the fjord is cut by the 50 foot beach. On the opposite side of the fjord a valley, at the head of which a glacier comes down from the Snée-fond, opens upon the shore, and is curtained across by a terrace, the surface of which is mottled with a number of irregular concentric mounds. We had not an opportunity of examining these, but they seemed to be moraine heaps left by the glacier when it came down to the fjord. They vividly recalled the singular concentric mounds that overlie the terrace at the mouth of the old glacier valley of the Brora in Sutherlandshire.

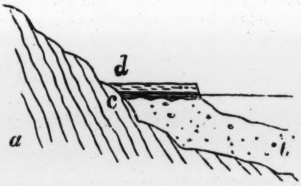
We walked along the north-east side of the fjord, and found the rocky declivity terraced with old sea-margins, which ran along like ancient and ruined roadways. They occur up to perhaps 200 or 250 feet above the sea-level, and are cut in the hard rock. They are covered with loose blocks, partly derived from the rocks around, but probably in part also transported from a higher part of the valley. On the beach we met with well ice-worn bosses of gneiss, slipping beneath a grey sandy clay full of arctic shells—a conjunction which is closely paralleled by one on the shores of Loch Fyne.

Fig. 9.—Section on beach at Nus Fjord.



d Sandy-grey clay, with *Tellina proxima*, *Saxicava rugosa*, *Astarte elliptica*, *Cyprina Islandica*, &c. *a* Ice-worn gneissose rocks.

Fig. 10.—Section on beach at Ardmarnock, Loch Fyne.



d Sandy-grey clay, full of *Tellina proxima*, *Astarte borealis*, *Natica clausa*, *Cyprina Islandica* (in fragments sometimes seven-twelfths of an inch thick) and other northern shells. *c* Finely striated red clay, without shells. *b* Boulder-clay. *a* Ice-worn gneissose rocks.

In each case the rocks are beautifully smoothed and grooved, and show that the ice which moulded them moved down the length of the inlet. To the north and east of the Jökuls Fjeld the ground becomes lower, and descends wholly below the snow-line. The hills that bound the Alten Fjord, instead of rising into serrated peaks, like the higher tracts to the south, have a well ice-worn aspect, and recall the hills of Cantyre, or the scenery of parts of the Hebrides. Indeed, the whole of this northern district of Norway, from the Alten Fjord to beyond the North Cape, has the smoothed outline which further southward is found only on the lower zone of the mountains.* It seems as if a sheet of ice, descending from the south, had overriden all the fjords here and the comparatively low hills between them, and had advanced northwards to the Arctic Sea.

In fine, this short excursion into the Northern part of Scandinavia, furnished us with abundant proofs that the glaciation of the west of Norway was produced by a mass of land-ice, of which the present glaciers are the representatives. It likewise confirmed, in a most impressive way, the conclusion which has gained ground so rapidly within the last few years, that the glaciation of the Scottish Highlands, as well as of the rest of the British Isles, is in the main the work, not of floating bergs, but of land ice. This conclusion may, indeed, be regarded as demonstrated beyond all cavil by the ice-marks of Norway. Much good work might be done by trying to work out a detailed comparison of the glaciation of the Scandinavian peninsula with that of this country. More especially would it be of importance to ascertain how far the glacial deposits of the two countries can be compared. Doubtless the drift-covered slopes of Sweden, and those of the east and centre of Scotland, must have many geological features in common. It will perhaps be found that some of the difficulties which our Scottish drift presents are explained by the more extensive deposits of the north, while the latter may likewise suggest new explanations of phenomena, supposed to be already sufficiently intelligible.

2. On the Third Co-ordinate Branch of the Higher Calculus. By Edward Sang, Esq.

The object of the paper was to point out that the theory of variables has a third branch, bearing to the Differential and Integral Calculus a relation somewhat analogous to that which the theory of Logarithms bears to Involution and Evolution.

In the theory of Functions there are three connected variable quantities, viz., the Primary, the Function, and the Derivative. When the relation between the primary and the function is given, the discovery of the derivative belongs to the Differential Calculus. When that between the primary and the derivative is given, the function is found by help of the Integral Calculus. And the general problem to find the primary when the relation between the function and its derivative is given, belongs to a third branch, which may be called the Calculus of Primaries.

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* We did not go further than Hammerfest, but the same contour is retained over the low, tame district that separates Hammerfest from the North Cape.

It was stated, in the course of the paper, that, by help of well-known artifices, problems involving the first derivative or the second derivative can be converted into problems of integration, and that thus the first two chapters of the Calculus of Primaries are, as it were, abstracted; but that, whenever the order of derivation is higher than the second, the equations cannot be transformed so as to be rendered integrable.

The following Gentlemen were duly elected Fellows of the Society:—

CHARLES MOREHEAD, M.D., F.R.C.P. Lond.
Professor DAVID MASSON.
DAVID DOUGLAS, Esq.

The following Donations to the Library were announced:—

Abstracts of the Proceedings of the Geological Society of London. No. 139. 8vo.—*From the Society.*

Bulletin de la Société Impériale des Naturalistes de Moscou. No. 2. 1865. 8vo.—*From the Society.*

Sitzungsberichte der königl. bayer. Akademie der Wissenschaften zu München 1865. II. Heft 1, 2. 8vo.—*From the Academy.*

Astronomical, Magnetical, and Meteorological Observations made at the Royal Observatory, Greenwich, in the year 1863. London, 1865. 4to.—*From the Observatory.*

Transactions of the Linnean Society of London. Vol. XXV. Part II. 4to.—*From the Society.*

List of the Linnean Society of London for 1865. 8vo.—*From the Society.*

Proceedings of the Royal Society of London. Vol. XIV. No. 79. 8vo.—*From the Society.*

Monthly Returns of the Births, Deaths, and Marriages, registered in the Eight principal Towns of Scotland. December 1865. 8vo.—*From the Registrar-General.*

Address of Lieut.-General Sabine, delivered at the Anniversary Meeting of the Royal Society, London, on 30th November 1865. 8vo.—*From the Author.*

Meteorologische Waarnemingen in Nederland en Zijne Bezittingen en Afwijkingen van Temperatuur en Barometstand op vele plaatsen in Europa Uitgegeven door het Koninklijk Nederlandsch Meteorologische Instituut 1861–1863. Utrecht. 4to. —*From the Utrecht Society of Arts and Sciences.*

L'appareil Épisternal des Oiseaux décrit. Par. P. Harting. Utrecht 1864. 4to.—*From the Utrecht Society of Arts and Sciences.*

Bijdragen tol de ontwikkelings der Zoetwater Planarien. Utrecht 1865. 4to.—*From the Utrecht Society of Arts and Sciences.*

Verslag van het Verhandelde in de Algemeene Vergadering van het provinciaal Utrechtsche genootschap van Kunsten en Wetenschappen 1862–63–64–65. Aanteekeningen 1860–61–62–63–64. Utrecht. 8vo.—*From the Utrecht Society of Arts and Sciences.*

Abhandlungen der Philosophisch-Philologischen classe der Koniglich bayerischen Akademie der Wissenschaften. Band X. Historischen classe. Band IX. Band X. Erste Abtheilungen. Munchen. 4to.—*From the Academy.*

Journal of the Statistical Society of London. Vol. XXVIII. Part IV. 8vo.—*From the Society.*

Proceedings of the American Philosophical Society. Vol. X. No. 73. Philadelphia. 8vo.—*From the Society.*